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ABSTRACT

Recent research suggests that many middle school students approach mathematical story problems with strategies that are not based on possible meanings for the operations, yielding success for one-step problems, but providing a weak background for approaching algebra story problems. This document reports the findings and the materials developed by a National Science Foundation (NSF) funded project to supplement textbook offerings and to give a greater emphasis to meanings for the operations. The materials were field tested with middle school students and revised after consultation with an advisory panel. Despite success in helping some students, the immature strategies are resistant to change for many middle schoolers. The document is divided into two sections. The first section is the NSF final project report, containing project identification information, a summary of the completed project, technical information including the projects background and procedures, and a list of the project personnel. The second section, "Project Materials," contains the developed materials. The preface in this section discusses the underlying assumptions employed in developing the materials, types of contextual settings used for applications of addition, subtraction, multiplication, and division, and uses of open-ended lessons. The lessons are presented under six headings, mainly by operation, identifying "meaning-centered" lessons for a given setting that should be studied before "application" lessons are undertaken. The six headings are: (1) Number Sense; (2) Addition; (3) Subtraction; (4) Multiplication; (5) Division; and (6) Situational or Open-ended. (MDH)

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LINKING MATHEMATICAL OPERATIONS AND THEIR APPLICATIONS

Larry Sowder, Project Director

Final Project Report
National Science Foundation
MDR-8850566

Center for Research in Mathematics and Science Education

College of Sciences
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FINAL PROJECT REPORT
NSF FORM 98A

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PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address
San Diego State University Foundation
San Diego, CA 92182

2. NSF Program
EHR-MDR

3. NSF Award Number
MDR-8850566

4. Award Period
From 5/1/89 To 10/31/91

5. Cumulative Award Amount
$183,188

6. Project Title
Linking Mathematical Operations and Their Applications

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

A considerable body of recent research suggests that many middle school students approach mathematical story problems, the most common school form of applications of the arithmetic operations, with strategies that are not based on possible meanings for the operations. Although some of these strategies may give success on many one-step story problems with whole numbers, their use with multistep problems or problems involving fractions or decimals is unlikely to give correct solutions. Furthermore, these immature strategies provide a weak background for approaching algebra story problems.

In this project, a team of experienced teachers and a university mathematics educator developed materials to give a greater emphasis to meanings for the operations. These materials were field tested with middle schoolers and then revised after consultation from an advisory panel made up of a psychologist, a mathematics supervisor, and a person sensitive to minority concerns. The tests and interviews during the project suggest that even though the materials seem to help some students, the immature strategies are quite resistant to change for many middle schoolers. Project materials, along with those developed in an earlier project, have been disseminated to text publishers to give them prototypes for such lessons in their textbooks.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM	
				Check (✓)	Approx. Date
a. Abstracts of Theses	X				
b. Publication Citations	X				
c. Data on Scientific Collaborators		X			
d. Information on Inventions	X				
e. Technical Description of Project and Results		X			
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*
Larry Sowder

3. Principal Investigator/Project Director Signature
Larry Sowder

4. Date
Jan 10, 1992

PART IV - SUMMARY DATA ON PROJECT PERSONNEL

NSF Division EHR-MDR

The data requested below will be used to develop a statistical profile on the personnel supported through NSF grants. The information on this part is solicited under the authority of the National Science Foundation Act of 1950, as amended. All information provided will be treated as confidential and will be safeguarded in accordance with the provisions of the Privacy Act of 1974. NSF requires that a single copy of this part be submitted with each Final Project Report (NSF Form 98A); however, submission of the requested information is not mandatory and is not a precondition of future awards. If you do not wish to submit this information, please check this box ☐

Please enter the numbers of individuals supported under this NSF grant.
Do not enter information for individuals working less than 40 hours in any calendar year.

*U.S. Citizens/ Permanent Visa	PI's/PD's		Post-doctorals		Graduate Students		Under-graduates		Precollege Teachers		Others	
	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.
American Indian or Alaskan Native												
Asian or Pacific Islander												
Black, Not of Hispanic Origin												
Hispanic												
White, Not of Hispanic Origin	1								1	3		
Total U.S. Citizens	1								1	3		
Non U.S. Citizens												
Total U.S. & Non-U.S. ..	1								1	3		
Number of individuals who have a handicap that limits a major life activity.												

*Use the category that best describes person's ethnic/racial status. (If more than one category applies, use the one category that most closely reflects the person's recognition in the community.)

AMERICAN INDIAN OR ALASKAN NATIVE. A person having origins in any of the original peoples of North America, and who maintains cultural identification through tribal affiliation or community recognition.

ASIAN OR PACIFIC ISLANDER. A person having origins in any of the original peoples of the Far East, Southeast Asia, the Indian subcontinent, or the Pacific Islands. This area includes, for example, China, India, Japan, Korea, the Philippine Islands and Samoa.

BLACK, NOT OF HISPANIC ORIGIN: A person having origins in any of the black racial groups of Africa.

HISPANIC: A person of Mexican, Puerto Rican, Cuban, Central or South American or other Spanish culture or origin, regardless of race.

WHITE, NOT OF HISPANIC ORIGIN. A person having origins in any of the original peoples of Europe, North Africa or the Middle East.

THIS PART WILL BE PHYSICALLY SEPARATED FROM THE FINAL PROJECT REPORT AND USED AS A COMPUTER SOURCE DOCUMENT. DO NOT DUPLICATE IT ON THE REVERSE OF ANY OTHER PART OF THE FINAL REPORT.

FINAL REPORT
PART III--Information on Project Collaborators
LINKING MATHEMATICAL OPERATIONS AND THEIR APPLICATIONS
MDR 8850566

Members of the Development Team

Larry Sowder, PI, Professor of Mathematical Sciences, San Diego State University

Tommie Jackson, Mathematics Teacher and Department Chair, Montgomery Junior High School, San Diego

Frances Peterson, Mathematics Teacher and Department Chair, Grant Middle School, Escondido, CA

Bonnie Schappelle, Mathematics and Reading Resource Teacher, Lincoln Prep High School, San Diego

Aileen Staples, Mathematics Teacher, San Marcos Junior High School, San Marcos, CA

FINAL REPORT
PART III--TECHNICAL INFORMATION 1e.
LINKING MATHEMATICAL OPERATIONS AND THEIR APPLICATIONS
MDR 8850566

Significance

The aim of this project was to provide textbook publishers with additional lessons emphasizing the uses of the arithmetic operations (addition, subtraction, multiplication, division), with the intent of encouraging an increased attention to such work in subsequent editions. Perhaps the main justification for including the arithmetic operations in the required curriculum is the need for an informed citizen to function in the many quantitative situations encountered in everyday life. Calculating skills are empty skills if they cannot be applied. Yet, children's performance on the most common school form of applications of arithmetic–the typical "story" problem–is dismaying when any degree of complexity is introduced (e.g., multiple steps, extra information). Recent work suggests that even a fairly good performance on one-step story problems may be tainted by the common use of ad hoc methods which have only limited applicability.

This project illustrates one way that research efforts can have an impact on textbooks. Gawronski has noted the gap between research findings and curricular change reflecting those findings when she asks "...where are the research-based teaching materials?" (1987, p. 2). Text authors no doubt often are aware of research articles, but these are not always easy to translate into text lessons. The project work addressed an apparent defect in the prevailing curricula, by designing and making available to publishers materials directed toward that defect.

Background

Because of their scope, the National Assessments of Educational Progress give the most sweeping view of how students perform on story problems in a group-test format. Analysts of one Assessment concluded, "Students appear to be learning many mathematical skills at a rote manipulation level and do not understand the concepts underlying the computation" (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980, p. 338). The test performance indicates that there is a poor link between carrying out computations for the operations and recognizing when the operations can be used. Furthermore, recent interview studies of students' strategies suggest that even students who are getting correct solutions on whole number story problems, let alone those involving rational numbers, may be using strategies of little long-term value.

Here is a fairly complete list of strategies that have been observed (cf. Sowder, 1988):

1. Find the numbers and add.
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "...if it's like, 78 and maybe 54, then I'd probably either add or multiply. But (78 and) 3, it looks like a division because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both + and x and choose the more reasonable answer. If smaller, try both - and + and choose the more reasonable.
7. Choose the operation whose meaning fits the story.

Note especially that strategies 4 and 6 may be effective with some whole numbers, but are less useful when large whole numbers, fractions, or decimals for which the student has less number sense are involved, and of course completely worthless with most algebra story problems involving operations on unknown numbers. All but the last strategy (#7) are extremely difficult to apply to multistep problems. What is most disappointing but pertinent here is the rarity of the meaning-based strategy 7. In interviews, even students who make correct choices of operations rarely can give any justification for their choice of operation.

A further example of the limits of students' meanings for the operations, and evidence of the influence of strategy 6 (decide whether the answer should be larger or smaller, if larger, try both + and x, etc.), is given by what has been called "non-conservation of operation" (Greer & Mangan, 1984): Students who correctly decide that multiplication (or division) will give the solution to a problem involving whole numbers may then change their minds when the same problem is given with fractions or decimals less than 1 in place of the whole numbers. The most reasonable explanation of this nonconservation phenomenon lies in the the unintended curriculum from the students' years of work with whole numbers: Multiplication makes bigger, division makes smaller (MMBDMS). For whole numbers that appear in one-step story problems, MMBDMS is all right and makes Strategy 6 a successful one, unless the numbers are so large that the student has little number sense for them. That this limited strategy continues even into college indicates that the curriculum is deficient in supplanting it, or better yet, in preventing its growth.

Text treatments are important. Even though many teachers do use the suggestions in the teacher's manuals and no doubt supplement the text presentations of the operations and story problems, it is probably true that what appears in the student's text is the focus of many lessons. If only a few illustrative examples are devoted to a meaning for an operation, they cannot be expected to have a robust effect and may even be inadvertently neglected. No doubt the many story problems throughout textbooks do add meaning to the operations for some students. If, however, the students cannot relate meanings to the operations, they will likely flounder and by default resort to strategies like the immature ones above, and thus not benefit from the potential embellishment of meaning from the work with story problems.

It is encouraging to find that an occasional, recent text series shows improvement in this area, primarily through attention to language for uses for the operations with whole numbers (e.g., divide to find out how many equal groups are in an amount). Nonetheless, it is still difficult to see much improvement in the provision of meaning for the operations with decimals and fractions. Many of the uses of the operations are the same, of course; totalling known decimal amounts still is modelled mathematically by addition, for example. But a new meaning is necessary when multipliers which are fractions (especially < 1) are introduced, and this topic continues to receive only token treatment. To summarize, examination of text series suggests that there is a shortage of material on the meanings for the operations, especially if one uses as a criterion the provision for student work with a variety of translations. Consequently, this project aimed to supply publishers with examples of additional materials supporting the development of meanings for the operations.

Procedure

The plan for the project included five phases: preliminary planning, materials development, try-out, revision, and dissemination. The PI recruited the Lead Teacher (Bonnie Schappelle), an experienced teacher with an interest in the project, and the rest of the Development Team (Tommie Jackson, Frances Peterson, Aileen Staples), other experienced middle school teachers who had been involved in special programs (two were department chairs). The PI and the Lead Teacher also updated their knowledge of the literature during the preliminary planning. (A rough copy of that literature search is available.)

Perhaps the most difficult decision for the Development Team was in choosing a framework for the meanings of the operations. There are several available, with many attractive features. Rather than adopt a framework that might be regarded as too avant garde by publisher, we settled largely on one by Greer (1987), perhaps also because the Development Team was able to meet with Greer, who is a psychologist with particular interest and expertise in story problem research.

His approach follows the now-common type of analysis for addition and subtraction but represents a refinement of the usual breakdowns of multiplication (into repeated addition, Cartesian product, part of an amount, scaling) and division (sharing, repeated subtraction). Greer, for example, distinguishes between symmetrical and asymmetrical types of multiplication (and division) problems. Asymmetrical multiplication occur in situations in which the multiplication is "psychologically noncommutative;" 3-sets-of-5 is not the same psychologically as 5-sets-of-3, even though 3x5 = 5x3. Greer's labels for the types are fairly self-explanatory. He calls the asymmetrical types multiple groups (the example just given), iteration of measure, change of scale, rate, and measure conversion. The symmetrical cases are labelled rectangular array, combinations (= Cartesian product), and area.

The Development Team devoted Summer, 1990, to the production of the first draft of the project materials, and met in January, 1991, to discuss the progress of the tryout. It is noteworthy, but understandable, that most of the teachers were quite unfamiliar with thinking about the operations in this way; this lack of familiarity certainly contains implications for teacher preparation and inservice, in dealing with applications of mathematics.

Nine classes of seventh or eighth graders in two schools were involved in the tryout, five using the materials and four not. One of two forms of a pretest was given early in the school year to each student, with the same form given in May. The tryout of the materials was unfortunately limited when the teaching assignments of two of the Development Team changed and they were no longer teaching classes for which the materials were appropriate. Based on the posttest results and teacher recommendation, two students from each class were selected and interviewed by the PI.

The written test results (Appendix B contains the posttest results for all students tested) and the interviews were somewhat disappointing in that the users of the project materials did not clearly outshine the control students. Some students seemed to have responded to the thrust of the materials, but others seemed to continue to use the immature strategies. The tryout teachers were surprised, since their perceptions were that the material was "taking" when they presented it to the classes. It may be that old habits are hard to break, and that a better approach is to emphasize meanings of the operations from the early grades on, rather than to try to correct bad habits in middle school. Two of the teachers did indicate an intent to emphasize the material even more than they had during the tryout year, so it will be interesting to see whether they are more successful.

The Development Team met with Advisory Panel members Dr. Sandra Marshall (psychologist) and Dr. H. Vance Mills (Mathematics and Science Program Manager for the San Diego City Schools) to gain their perspectives and suggestions for use during the revisions. (Dr. Frank Holmes, also an Advisory Panel member and a minority expert, was out of town, with his input planned later.) The meeting was quite worthwhile, with the Panel suggesting emphases and directions to consider during the revision.

Based on the tryout experiences, the testing and interview results, and the input from the Advisory Panel, the Development Team spent one month during Summer, 1991, revising and extending the materials.

Product and Dissemination

The final form of the project materials is included as Appendix A. These materials, along with the materials developed during MDR 8696130, have been sent to publishers of junior-high mathematics texts, and will be sent with this final report to ERIC for consideration. On an at-cost basis, they will also be available to any other interested party.

References

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APPENDIX A

PROJECT MATERIALS

LINKING
THE MATHEMATICAL OPERATIONS
WITH THEIR APPLICATIONS

Development Team
Tommie Jackson, Montgomery Junior High School, San Diego
Frances Peterson, Grant Middle School, Escondido
Bonnie Schappelle, Lincoln Preparatory High School, San Diego
Larry Sowder, San Diego State University, San Diego
Aileen Staples, San Marcos Junior High School, San Marcos

Summer, 1990, Version

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Table of Contents

Preface

The Lessons

Notes on the Lessons:
- Lesson titles (and page numbers) are given in the body of the table.
- "Meaning-Centered" lessons for a given setting should usually be studied before "Application" lessons for that setting are undertaken.
- The lessons are organized mainly by operation, but number sense is regarded as essential for understanding the operations. Multiplication and division lessons may also involve addition and subtraction. Lessons later in the list may use ideas from earlier ones. Situational lessons, calling as they do on a variety of operations and a variety of settings, are listed separately.
- Within a set of lessons, titles are alphabetical and no order is implied, except as noted. The "Student Background" in the notes for a lesson may suggest prerequisite lessons.
- Equal-groups-or-amounts and part-of-a-group-or-amount multiplications are basic in most curricula and should, as a rule, precede other multiplication and division lessons.

	Meaning-Centered	**Application**
Number Sense		
Fractions	Fraction Sense (p. 1) (9 parts)	
Decimals	Decimal Sense (p. 30) (5 parts)	
Percents	Percent Sense (p. 52) (5 parts)	
Ratios	Ratio Sense (p. 77) (4 parts)	
Addition	Putting It Together (p. 84)	Adding and Subtracting Fractions (p. 94)
Subtraction		
Take-away	Take It Away & Compare (p. 88)	
Comparison		

Multiplication		
Equal groups/amounts	Is That True? (p. 99)	Pizza Party (p. 102)
Part of group/amount	Fractional Parts (p. 107) Parts or Groups (p. 109)	The Pay-Off (p. 113)
Special cases of above	High School Enrollment (p. 116) Find Area & Volume (p. 121) What's in an Area? (p. 127)	
Rates	At Any Rate (p. 132) Pulse Rate (p. 136) (3 parts)	Can a Marble Break...(p. 151) Endurance & Math... (p. 152) Long Distance, Please (p. 156) Long Mower (p. 160) Plan Ahead (p. 162) Want Ad (p. 166)
Comparing with multiplication		
-"times as much"	Let's Compare (p. 168) Vernal Falls (p. 173)	Chili Cookoff (p. 176) Universal Sports (p. 184) Pulse Rate (p. 136)
-similarity	New Bedroom (p. 192)	Create a Hollywood Monster (p. 198)
Making choices		
-Cartesian product	New Clothes (p. 215)	Decisions, Decisions (p. 218) Auto Options (p. 221)
-fundamental counting principle	Exclamation! (p. 227)	
Formulas with multiplication	Find Area & Volume (p. 121)	Hit by a 10-Ton Truck (p. 232)
Division		
Sharing equally	}Chili Cookoff (p. 176) }It's Only Fair (p. 234)	Wise Shopper (p. 244) }
Repeated subtraction	}Easy As Pie (p. 237) (should follow It's Only Fair)	}Disneyland (p. 247) Chili Cookoff (p. 176)
Missing factor		Missing Factors (p. 250) Universal Sports (p. 184)

Situational or Open-ended

A Worldly Trip (p. 253)
Best-Buy Thirst Quencher (p. 255)
Don't Use Your Brain (p. 257)
Fast Food, 2 parts (p. 258)
Global Conservation (p. 271)
Newsworthy Problems (p. 274)
Open-ended Problems (p. 276)
Playground (p. 278)
Public Smoking (p. 280)
Student Survey (p. 283)
The Question Is (p. 285)
This Is Your Life (p. 289)
Water Conservation (p. 296)
Worst T-Shirt Ever (p. 299)

Preface

Overview

This project represents one response to the demands of curricula which are being shaped by such documents as the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989). Such recommendations recognize both the richness of mathematics and the reasonableness of moving away from a curriculum centered on computation, with calculators easily available.

One dimension of the proposed changes is to increase the attention to applications of mathematics, both to motivate students and to increase their mathematical power. As a part of such endeavors, "...teachers should emphasize the application of mathematics to real-world problems as well as to other settings relevant to middle school students" (NCTM, 1989, p. 66). The activities written for this project are intended to supplement textbook offerings along this line. Research unfortunately indicates that many students approach story problems somewhat mindlessly, in some cases because their "concepts" of the operation reside solely in how one calculates for that operation. Hence, the project activities centerpiece additional meaning-center lessons. Although story problems are the most common school form of applications of mathematics, problems which are much more open ended than those in the usual lists of story problems are also included, under the rubric "situational lessons" (cf. California State Department of Education, 1985).

Assumptions

The project lessons recognize the importance of such Standards thrusts as estimation, mental arithmetic, number sense, mathematics-as-communication,.... Consequently, some lessons focus on number sense because of its importance in understanding applications of the operations. And there are frequent suggestions for work in groups and for written or oral reports by individuals or groups. Two assumptions must be highlighted.

- In the large, project lessons assume that at least some **calculators are available**. Truer-to-life numbers can then be used in problems, with calculations that would be intimidating with paper and pencil no longer an obstacle.
- Rather than have the students directly write an appropriate calculation expression, most project lessons assume that **students will write an equation to represent the problem.** This assumption means that attention to solving simple equations should be included.

What types of applications are included?

There are several excellent ways of analyzing the applications of the arithmetic operations (e.g., Greer, in press, Kaput, 1985, Marshall, 1990, Usiskin and Bell, 1983). We settled on a breakdown that amplifies the "traditional" one, primarily so that we could build on what students are likely to have seen in earlier grades. The lessons give some attention to addition and subtraction settings but assume that the many difficulties that younger students have with these two operations are no longer prominent. Rather, most of the lessons focus on applications of multiplication and division. Following is the breakdown we used, with a few illustrative story problems.

Types of Addition Settings

Groups or amounts are put together, either physically or conceptually.

A. The attendance at three baseball games was 28,542, 11,451, and 17,569. What was the total attendance at those three games? (Groups are put together conceptually.)

B. The scientist mixed 4.3 liters of solution A and 1.6 liters of solution B. If the solutions do not react, what is the volume of the mixture? (Amounts are put together physically.)

C. You earned a total of $24.50 doing odd jobs for two neighbors. One neighbor paid you $7.75. How much did the other neighbor pay you? (Amounts are put together physically; note that the missing-addend equation, 7.75 + x = 24.50, reflects the situation in the problem rather than the computation required to answer the question.)

Types of Subtraction Settings

1. An amount (or group) is removed from another amount (or group) [take-away subtraction].

D. The cook bought 36 dozen eggs and used 28 eggs in one recipe. How many eggs did the cook have left?

2. Two distinct amounts are compared in a how-much-more or how-much-less sense [comparison subtraction].

E. Magic Johnson is 6' 8" tall. How much taller (or shorter!) is Magic than you are? (Note: The "correct" equation is 6' 8" - (your height) = x. Although there are two separate amounts being compared here, one can conceive of the situation as a missing-addend one: (your height) + x = 6' 8". If a student who writes a missing-addend equation for a comparison subtraction can **justify** the equation, we would be delighted.)

Types of Multiplication Settings

1. Equal groups or amounts are put together.

F. How many seats are there in 30 rows if each row has 42 seats? [By far the most common convention in US textbooks is to give the number of groups or amounts as the first factor. Under this convention, this problem would be described as 30 x 42, not 42 x 30. Students may already be comfortable with commutativity of multiplication and know that 30 x 42 and 42 x 30 give the same answer, but they may not know or observe the convention. The important thing is that they can justify what they write.]

G. How much do a dozen 1.6-ounce candy bars weigh?

2. Part of a group or amount is described.

H. The test had 125 points on it, and you got 84% of them. How many points did you get? [The "'part of' means multiply" reading leads to 84% x 125 = x.]

I. The test had 125 points on it, and you got 104 of them. What percent was that? (This example is to illustrate that the equation x125 = 104 (or 125x = 104) could be used.)

J. Although the miner had collected 122.5 grams of gold, he gave his helper only 1/8 of it. How many grams did his helper get?

[Note: Mixed-number multipliers can be viewed as a combination of the equal-groups-or-amounts and the part-of-group-or-amount settings. For example, the cost of 4.6 pounds of apples at $0.79 per pound could be viewed as the cost of 4 pounds of such apples (equal-amounts multiplication), plus the cost of 0.6 pound of such apples (part-of-amount multiplication). Area and volume formulas can also be viewed as originating in these two kinds of multiplication settings.]

3. Rate settings in general, when the rate is given per unit, can be described by multiplication equations: (m Unit 1) x (n Unit 2 per Unit 1) = (mn Unit 2)

K. The d = rt relationship describes a familiar "rate" situation:
(r miles/hour) x (t hours) = (d miles)
About what is your average speed (in miles per hour) if you ride 11.2 miles in 55 minutes on your bike?

L. How much will 8 packages of cupcakes cost if the cupcakes costs 69¢ per package? [This example illustrates that equal-groups-or-amounts multiplication could be subsumed under rates:
(m groups) x (n items/group) = (mn items). We did not press this subsumption because of the students' likely experience in earlier grades with the equal-groups-or-amounts view, commonly called repeated addition.]

4. Distinct quantities are compared (a) as in a times-as-many-or-much sense, or (b) as in a setting where the quantities may be related by a proportion.

M. The population of the city is now 125% times as much as it was in 1980. Its population in 1980 was 125,000. What is its population now?

N. If a pet store sells 5 kittens for every 2 rabbits, how many kittens does it sell when it sells 30 rabbits? (Note that by thinking of the unit rate 2.5 kittens/rabbit, this problem could also be classified as a rate problem.]

O. If the scale factor on a map is 5000:1, what length does 3 cm on the map really represent?

5. Cartesian product settings, and settings to which the fundamental counting principle can be applied, may be described by multiplication [making-choices multiplication].

P. Ronnie R. Thomas can write the letter "R" in 3 ways and the letter "T" in 5 ways. In how many ways can Ronnie write the initials "RT"?

Q. In how many ways can Ronnie R. Thomas write the initials "RRT" without repeating the style of "R" used?

R. Ann, Ben, Cid, and Don have been chosen for a student committee. The committee will have a chair, an assistant chair, a secretary, and a treasurer. In how many ways can the students fill those positions?

6. Some settings involve a multiplication through a definition or through a scientific relationship.

S. If you burn a 100-watt bulb all the time, how many kilowatt-hours does the bulb use in May?

T. What force causes a 2-kilogram mass to accelerate at 20 m/s? [F = ma is a basic equation from science. Project lessons do not involve such unfamiliar formulas without explanation; this category is included in an effort to be somewhat complete.]

Types of Division Settings

1. Situations in which a group or an amount is distributed into a known number of equal groups or amounts may be described by division [sharing-equally division].

U. The school has a supply budget of only $500 for 12 teachers. How much money will each teacher be allotted if each gets the same amount?

V. A large 6.84-pound package of hamburger is to be used for four meals. How much hamburger does that allow for each meal?

W. There are six people in the family using that hamburger. How much hamburger does each person get at each meal?

2. Situations in which a group or amount is put into groups or amounts of a known size may be described by division [repeated-subtraction division].

X. The principal decides that each teacher should have $50 in supply money. How many teachers are covered by a supply budget of $500?

Y. How many 4-ounce patties can be obtained from a 6.84-pound package of hamburger?

Z. How many 2' 10" shelves can a carpenter get from a board 8' long?

3. Situations involving a missing factor may be described by a division equation. Both 1 and 2 above may be thought of in multiplication equations with a missing factor.

AA. A rectangle has length 4.2 cm and an area of 2.1 cm^2. What is its width?

U'. Problem U above might be thought of as $12 \times a = 500$.

X'. Problem X above might be thought of as $n \times 50 = 500$.

Situational and Open-ended Lessons

These types of lessons are increasingly recognized as being important. A curriculum in which the applications all reside in three- or four-sentence story problems with all necessary information provided does not allow a student to develop a "I can do this" attitude toward a novel, ill-defined situation--like that most often encountered in a situation or open-ended lesson.

There are two rather different ways to use such lessons. The traditional way has been to expose the students to all the skills and ideas needed, as can best be predicted, before a situational lesson is assigned. There are many advocates of a second way: Use a situational lesson as a springboard for introducing a need for particular pieces of mathematics; then provide the needed instruction. In either case, adapting a given lesson to fit the local situation seems most likely to add relevance to the theme.

References

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FRACTION SENSE

FOCUS: Fraction Sense

PURPOSE: The student will . . .

- Review the meaning of a fraction, using the part-whole, part of a set, and number line models;
- Review fraction terminology;
- Review equivalent fractions;
- Review the importance of the unit;
- Learn to view a fraction as a quotient; and
- Review operations with fractions intuitively and with drawings.

STUDENT BACKGROUND: Students' knowledge of fractions will vary. The teacher will judge the extent of need for these lessons. Some exercises require knowledge of metric units and experience with number lines.

TEACHER BACKGROUND: This lesson, as needed, precedes the introduction of the part-of-a-whole meaning of multiplication and other operations using fractional numbers.

MATERIALS:

Part 1 Game: COOPERATION: Make copies of the pieces for the version chosen (one of each figure for each group), and cut the shapes into the fractional pieces indicated. Each group is to receive the pieces for one set of completed figures, but with the pieces for each figure distributed among envelopes for the group (i.e., Each student receives one envelope with parts to various figures).

Part 2 FRACTION: PART OF A WHOLE worksheet; scissors and copies of the enlarged figures in Problems 11 and 12 (optional).

Part 3 FRACTION: PART OF A GROUP worksheet.

Part 4 FRACTIONS ON A NUMBER LINE worksheet.

Part 5 EQUIVALENT FRACTIONS worksheet.

Part 6 MORE COMPARING FRACTIONS worksheet.

Part 7 A FRACTION AS A QUOTIENT worksheet.

Part 8 ADDING AND SUBTRACTING FRACTIONS AND MIXED NUMBERS worksheet; rectangular paper strips (Problem 7) - 3 per student.

LESSON DEVELOPMENT:

Part 1

Introduction: This review will focus on the meaning of fractions vs computational aspects.

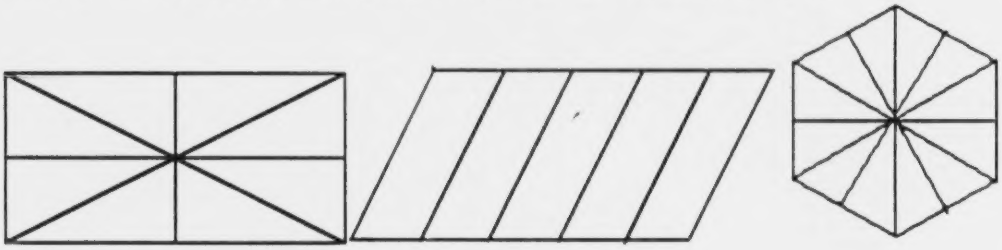
1

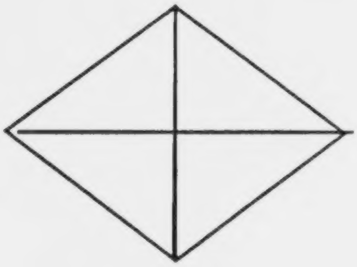
Game -- **Cooperation**:
Masters are included for the figures for two versions of this game. The versions differ only in the completed figures and their parts. In Version 1, the completed figures are all squares of the same size, but are made up of parts of different sizes. In Version 2, the students form different figures, but each figure consists solely of the same unit-fractional parts.

Students work in groups of four. Each group member receives an envelope containing fraction pieces, some for each figure to be completed within the group. The goal is for each student to complete one geometric figure; the pieces needed are to be given by group members. Talking is not permitted. A student may obtain a piece **only** if it is offered to him by a member of his group.
(If not all groups have four members, remove parts for one of the figures and distribute parts for three to group/s of three.)

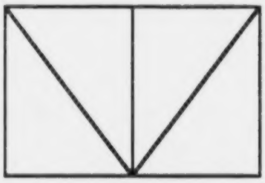
Upon completion, students will give the fractional name/s for the parts of their figures. The definition of a fraction n/d as n of d equal parts should be reiterated during the discussion following the game.

Among the possible completed figures for Version 2 are:





or



Parts 2--8
Worksheets provide exercises that students may use to explore varied models of fractions and to review fraction terminology. It is expected

that students' answers will be discussed in class. The explanations students give should include the terminology and meanings for fractions. For example, in explaining the reason a fraction in Part 1, Problem 1 is 1/3, students should explain that the shaded part of is 1 of **3 equal** parts. Exercises are designed to allow students to review equivalent fractions, improper fractions, the quotient interpretation of a fraction, and addition and subtraction of fractions using the models, without resorting to algorithms.

Part 5: Problem 3 does not ask students to explain their answers in writing, but they might be asked to explain orally, demonstrating that the answers may be reached in more than one way. For Problem 6, a different student could locate each fraction in turn on the number line reproduced on a transparency or drawn on the board. The names for a given point could be listed beside that point, since lines from the various names on the worksheet are difficult to follow to their origins.

ANSWERS:

Part 2 FRACTION: PART OF A WHOLE

1. a) No Pieces not equal
   b) Yes It is one of 3 equal parts.
   c) No Pieces not equal
   d) No Not 3 pieces
   e) Yes, even though the other dividing mark is not drawn.
   f) Yes, but some students may have trouble ignoring the "extra dividing marks.
   g) No Pieces not equal
   h) Yes

   One-third means one of 3 equal parts of a whole.

2. a) Answers vary
   b) Answers vary
   c) 6 fractions: $\frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{3}{5}, \frac{3}{7}, \frac{5}{7}$

3. Largest $\frac{1}{2}$; Smallest $\frac{1}{8}$;

   The fraction with the larger denominator is smaller. The fraction with the larger denominator is one of a larger number of equal parts of the same whole, so each piece is smaller.

4. $\frac{1}{3} > \frac{1}{4}$ $\frac{1}{5} > \frac{1}{7}$ Yes

5. $\frac{1}{2}$, $\frac{1}{3}$ ($> \frac{1}{4}$) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$ (not $< \frac{1}{10}$)

6. Three-fifths means **3** of **5** **equal** parts.

Largest $\frac{3}{4}$, Smallest $\frac{3}{8}$

For two fractions with the same numerator, the fraction with the larger denominator is smaller. Each part of the fraction with the larger denominator is smaller, so an equal number of these smaller parts makes a smaller sum.

7. The fraction with the larger numerator is larger since it has more pieces and the pieces are all the same size.

8. a) Shading --> $1\frac{1}{2}$
   b) 7
   c) 3 equal parts; 8 total parts are shaded. $\frac{8}{3} = 2\frac{2}{3}$
   d) 1, 2, 3, 4, 5, 6, 7, 8

9. Some of the many ways this can be done are suggested by other exercises in the lesson.

10. a) Yes Each is $\frac{1}{4}$
   b) Yes $\frac{2}{8} = \frac{1}{4}$
   c) No Parts unequal, same number of parts shaded.
   d) Yes Each is $\frac{1}{6}$
   e) Yes $\frac{1}{2} = \frac{1}{2}$
   f) Yes $\frac{1}{8} = \frac{1}{8}$
   g) Yes $\frac{1}{3} = \frac{1}{3}$

Note: The figures for Problems 11 and 12 are provided on a separate sheet in a size appropriate for folding in case students need to use them.

11. $\frac{1}{2}$ Hint: Fold each corner to the center along the sides of the inside square.

12. $\frac{1}{8}$ Hint: Find the part in the tangram that is $\frac{1}{16}$ of the whole. What part of the small square is it?

Part 3 FRACTION: PART OF A GROUP

1. 9 stars shaded; 6 hearts shaded
2. $\frac{2}{3}$ of 15 = 10 > 9 = $\frac{3}{4}$ of 12
3. Any 4 shapes shaded
4. 4/9
5. 14/29 About 1/2
6. a) = b) < c) < d) >
7. a) 30 15 20

b) 12 6 4
c) 500 250 125

Part 4 FRACTIONS ON A NUMBER LINE

3. a: 1/4 b: 3/4 x: 1 1/4 or 5/4 y: 1 2/4, 1 1/2, or 6/4

4. a) > b) = c) < d) > e) <
5. a) 1/2 b) 0 c) 1/2
6. $\frac{1}{5} > \frac{1}{10}$ $\frac{2}{5} > \frac{3}{10}$

Part 5 EQUIVALENT FRACTIONS

1. $\frac{1}{5} = \frac{2}{10}$
3. 1: 1/1, 4/4 (shown either by triangles or rectangles), 8/8
   1/2: 3/6, 2/4, 6/12
   2/3: 4/6, 8/12, 16/24
4. $\frac{2}{3} = \frac{6}{9}$ $\frac{2}{10} = \frac{1}{5}$ $\frac{1}{4} = \frac{4}{16}$ $\frac{3}{4} = \frac{6}{8}$
5. $\frac{2}{5} > \frac{1}{3}$ $\frac{3}{5} = \frac{6}{10}$ $\frac{3}{4} < \frac{4}{5}$
7. a) 8/16 = 1/2 b) 10/16 = 5/8 c) 4/16 = 1/4
8. a) Shade 12 b) Shade 6 c) Shade 3

I divided the large triangle into **4 equal parts** with 4 small triangles in each part. I **shaded 3 of the parts**, a total of 12 small triangles.

Part 6 MORE COMPARING FRACTIONS

1. $\frac{3}{4} < \frac{4}{5}$ of any unit.
3. 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 8/8
4. 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9

Part 7 A FRACTION AS A QUOTIENT

2. 3/4
3. 2/3 = 2 ÷ 3
4. 1 1/2
   3 ÷ 2 3/2 1 1/2
5. a) 4/5 b) 8 ÷ 3; 8/3; 2 2/3

Part 8 ADDING AND SUBTRACTING FRACTIONS AND MIXED NUMBERS

1. 2 + 1/3 2 + 3/8
2. 3 3/4 3 1/3
3. 1/3 1 - 2/3 = 1/3 or 2/3 + 1/3 = 1
4. a) 3/4 or 3/4

b) 1 5/8 or 1 5/8

5. 1/4 + 1/2 + 1/4 = 4/4 = 1 3/8 + 1/8 + 4/8 = 8/8 = 1
6. Less
7. 1/3 + 1/6 = 1/2
8. 16
   a) 2/2, 2/3, 2/5, 2/7, 3/2, 3/3, 3/5, 3/7, 5/2, 5/3, 5/5, 5/7, 7/2, 7/3, 7/5, 7/7
   b) No; 2/5 and 3/7
   c) Largest: $\frac{7}{2}$; Smallest: $\frac{2}{7}$
   d) Greatest sum: 7/2 + 5/2
   e) Smallest sum: 2/7 + 2/5

SOURCE: Parts of these lessons were adapted from the following:

Cervakne, E. N., Halmos, I., Szendrei, J., Radna, S., & Varga, T. (1985). *Mathematics workbook for Grade 4*(4th ed.). Budapest, Hungary: Tankonyvkiado.

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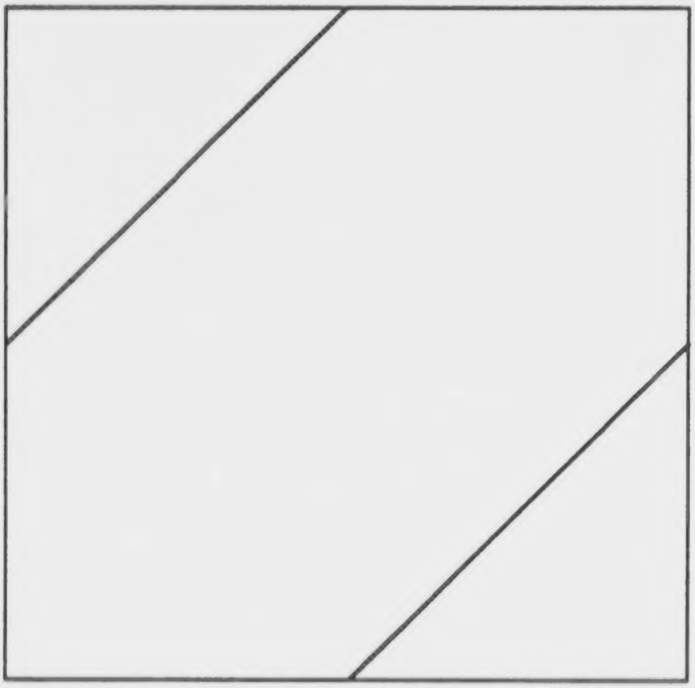
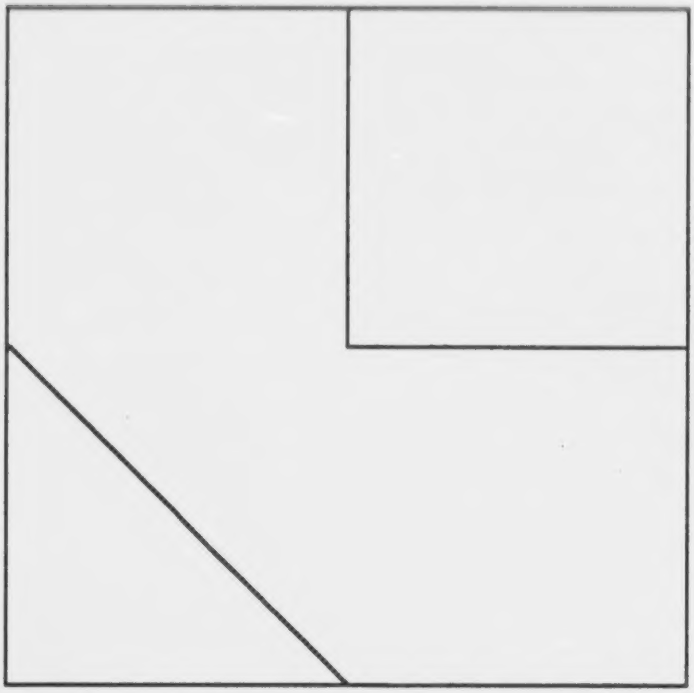
COOPERATION: Figures for Version 1 (Page 1 of 2)





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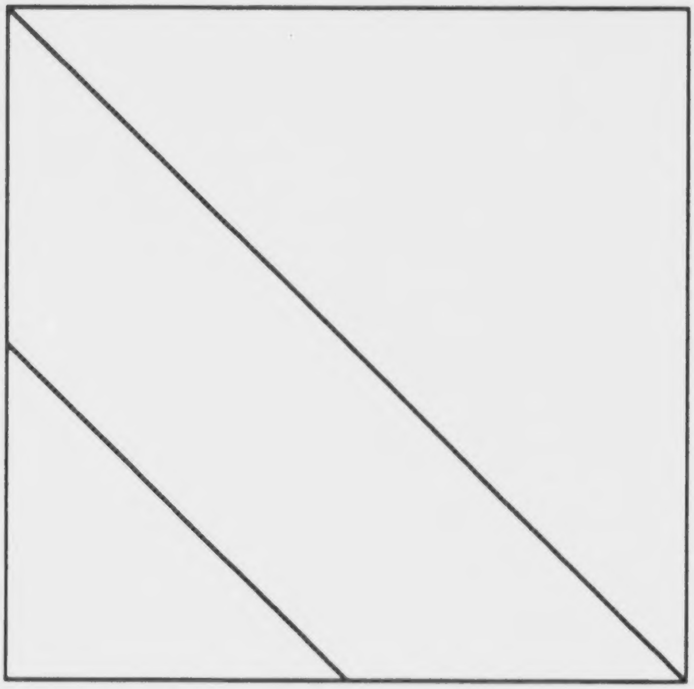

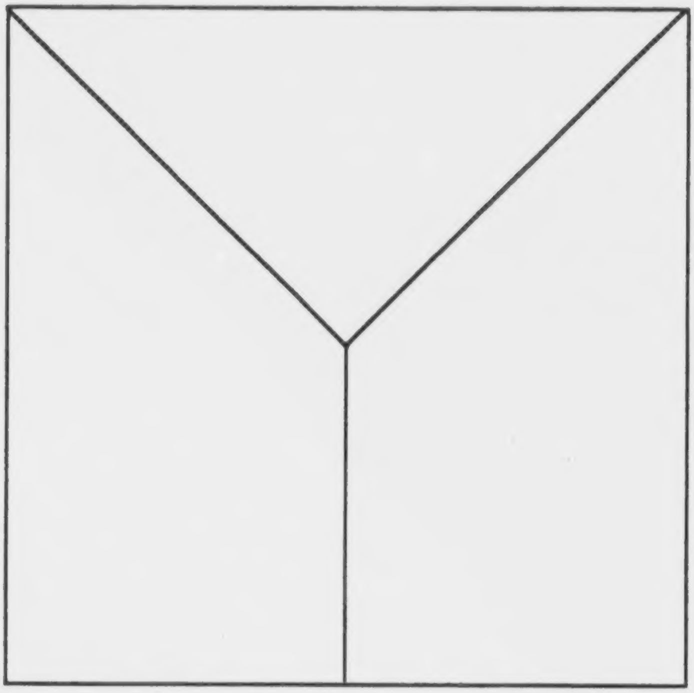

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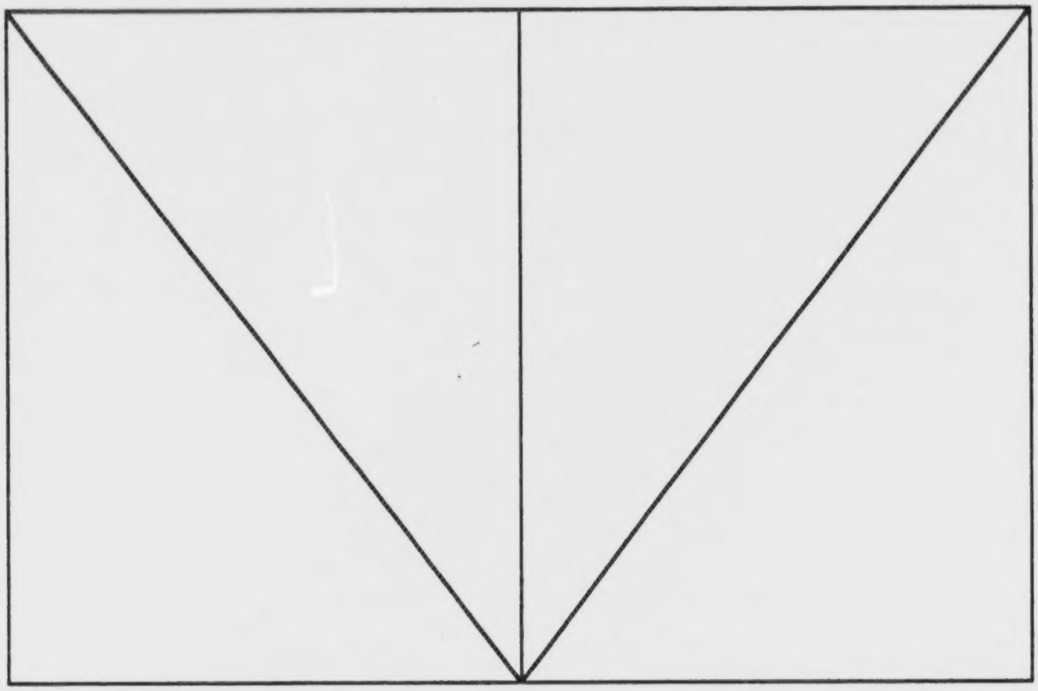

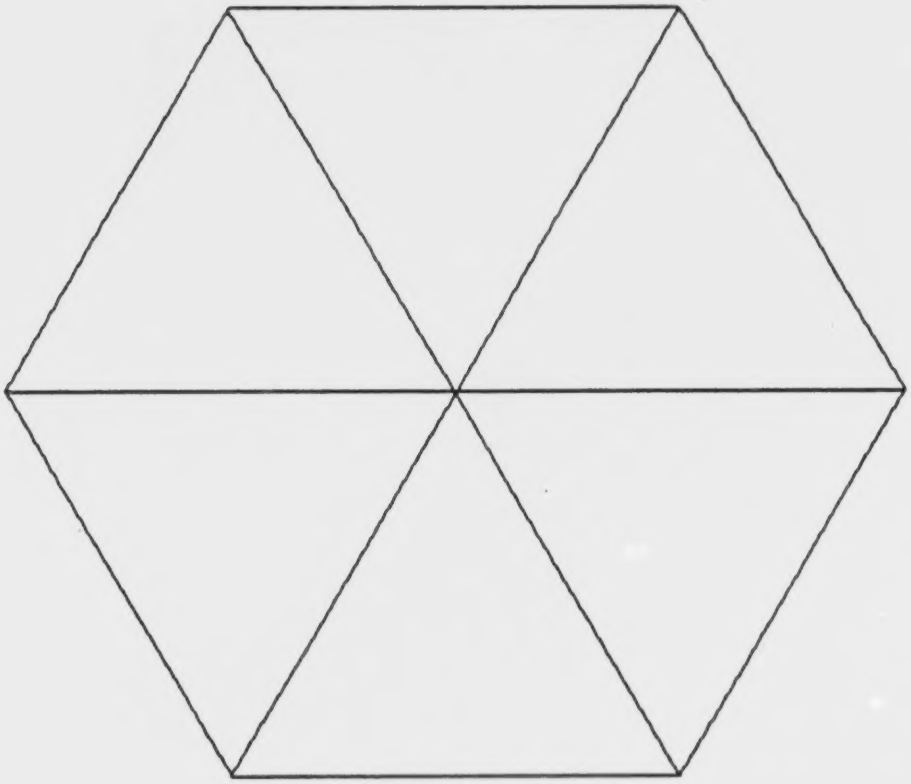

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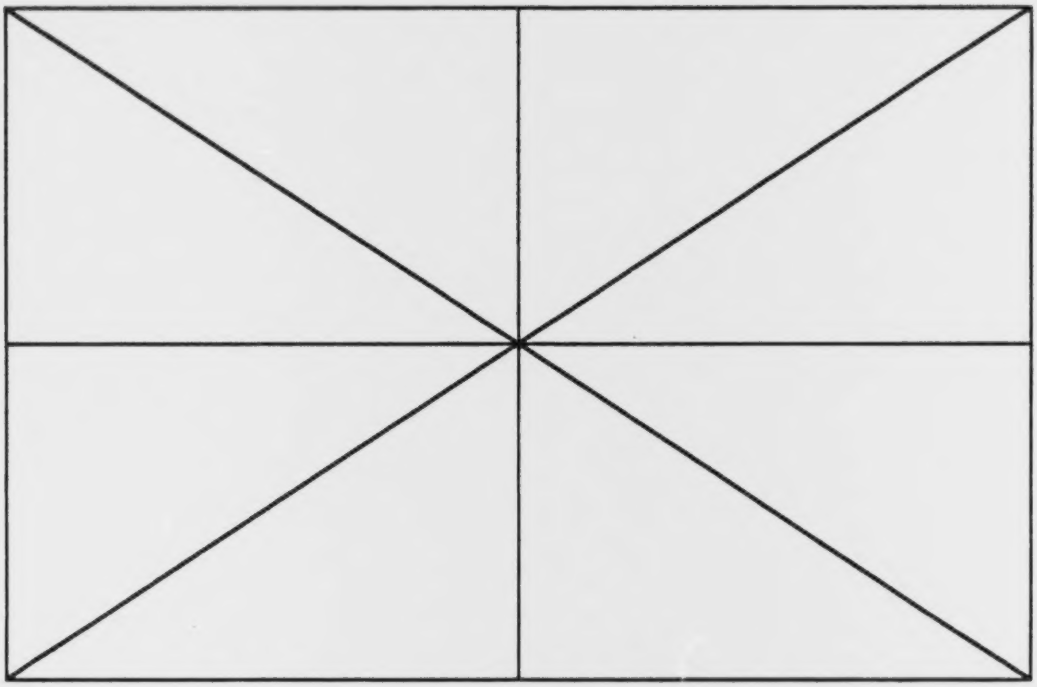
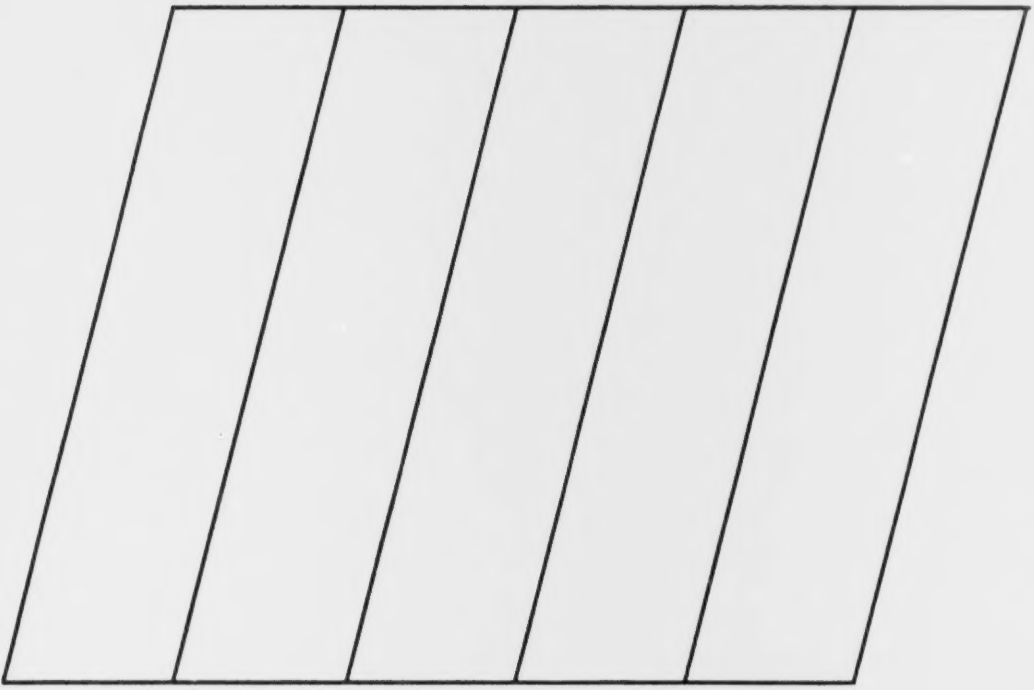
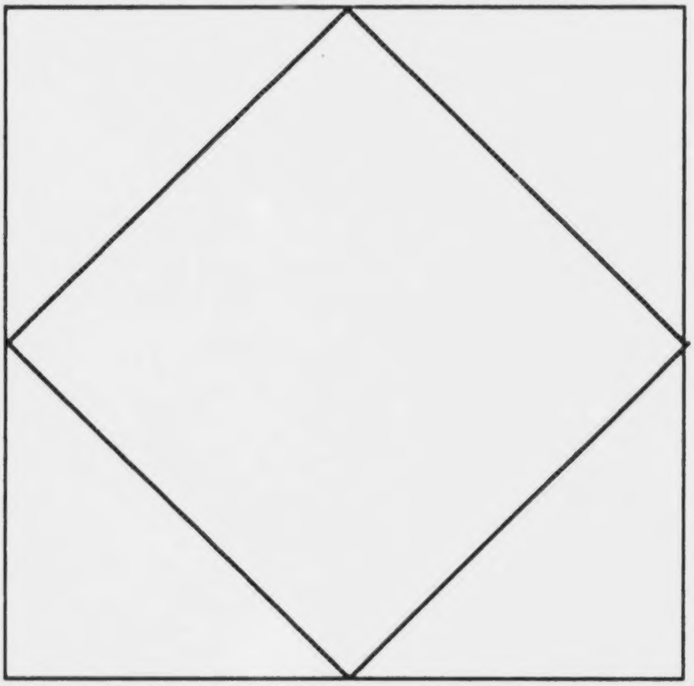


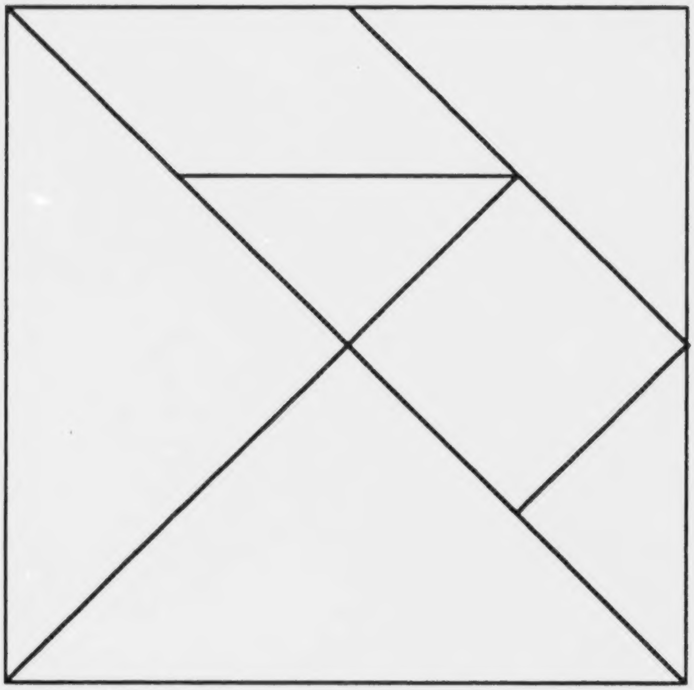


Figures for use in paper folding: Fraction Sense: Part 2

11.

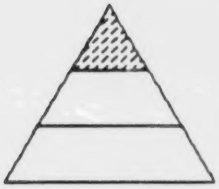


12.

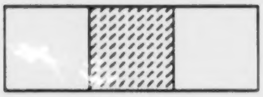


FRACTION: PART OF A WHOLE

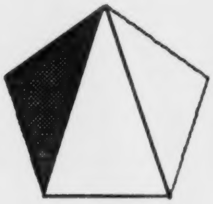
1. For each of the following figures, write YES if $\frac{1}{3}$ is shaded; write NO if not.



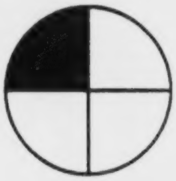
a) ____________



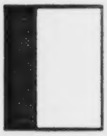
b) __________



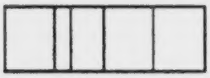
c) _______



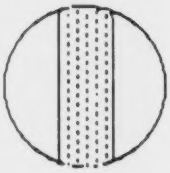
d) __________



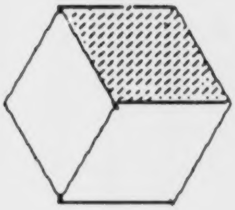
e) ____________



f) __________



g) _______



h) __________

Give the reason for each NO answer you gave.

Figure	Reason
_____	____
_____	____
_____	____
_____	____
_____	____
_____	____

The fraction one-third means ______________________________

2. Fraction vocabulary review

a) Name three fractions that have denominator greater than 5 and numerator less than 6.

b) Think up five fractions with numerator 3 less than the denominator.

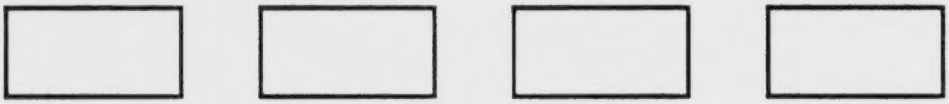
c) How many different fractions with a single-digit numerator less than the single-digit denominator can be formed using the following cards?

____

$\boxed{2}$ $\boxed{3}$ $\boxed{5}$ $\boxed{7}$

List the fractions.

3. Shade the indicated fraction of each of the following rectangles.



Shade $\frac{1}{2}$ Shade $\frac{1}{3}$ Shade $\frac{1}{4}$ Shade $\frac{1}{8}$

Which fraction is the largest? ___________

Which fraction is the smallest? ___________

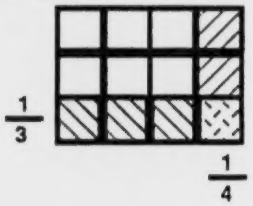
Explain how to compare the sizes of fractions with **numerator of one**. Explain why your method works.

____

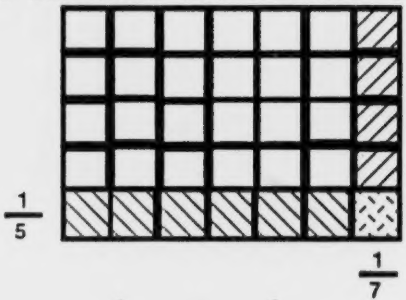
____

____

4. Compare the shaded parts to determine which number is bigger.



$\frac{1}{3}$ ☐ $\frac{1}{4}$

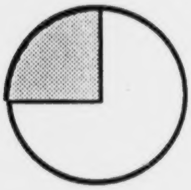


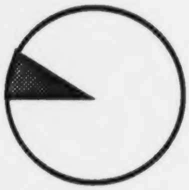
$\frac{1}{5}$ ☐ $\frac{1}{7}$

Does your method in Problem 3 give the same result? _______

5. Write the fractions that satisfy each statement:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{14}$ $\frac{1}{16}$ $\frac{1}{18}$





________________ $> \frac{1}{4}$ ________________ not $< \frac{1}{10}$

6. One third means 1 of 3 equal parts.
Three-fifths means ________________.

Shade the indicated fraction of each of the following rectangles.

Shade $\frac{3}{8}$

Shade $\frac{3}{4}$

Shade $\frac{3}{5}$

Which fraction is the largest? __________
Which fraction is the smallest? __________

Explain how to compare the sizes of fractions with the **same numerator.** Explain why your method works.

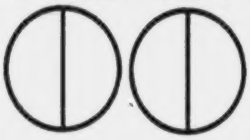
____
____
____

7. How do you compare fractions that have the **same denominator**? Use drawings to support your answer. Explain why your method makes sense.

____
____
____

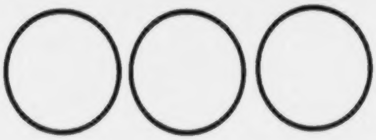
8. Fractions greater than 1

a) Shade the circles to show three halves.



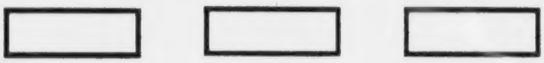
Write $\frac{3}{2}$ with a whole number part and a fraction part (a **mixed number**)._____

b) Shade these circles to show $2\frac{1}{3}$.



Using the figures, you can see than $2\frac{1}{3} = \frac{\square}{3}$.

c) Shade $\frac{8}{3}$.



To shade $\frac{8}{3}$, each whole is divided into _________ equal parts and _______ total parts are shaded.

Write the mixed number: $\frac{8}{3}$ = ______.

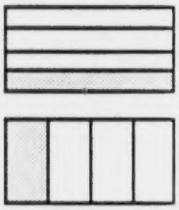
d) Give all values of x for which $\frac{9}{x}$ is an improper fraction.

9. Show as many ways as you can to shade $\frac{1}{4}$ of a rectangle.

10. For each pair of figures, tell whether the shaded areas are equal. Justify each answer. Assume that the two figures in each part are the same size.

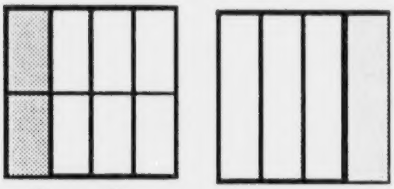
Explanation

a)



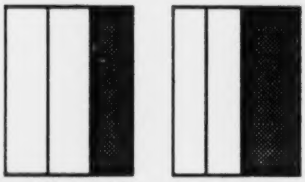
Equal? _____

b)



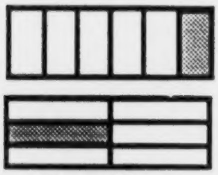
Equal? _____

c)



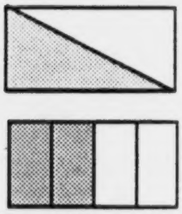
Equal? _____

d)



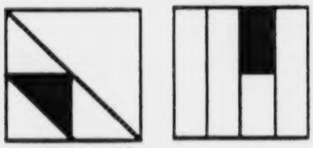
Equal? _____

e)



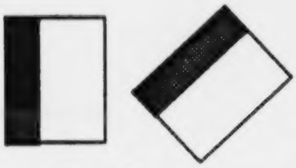
Equal? _____

f)



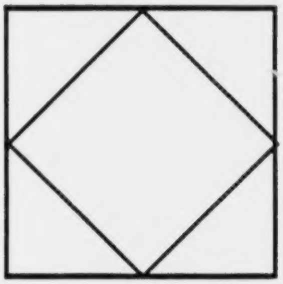
Equal? ______

g)

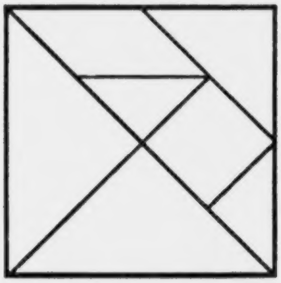


Equal? ______

11. The small square (the inside square) is what part of the larger square (the outside square)? __________



12. The small square in this tangram puzzle is what fractional part of the larger square? _______



FRACTION: PART OF A GROUP

1. Shade $\frac{3}{4}$ of the stars: Shade $\frac{2}{3}$ of the hearts:

☆ ☆ ☆ ☆
☆ ☆ ☆ ☆

♡ ♡ ♡
♡ ♡ ♡
♡ ♡ ♡

Three-fourths of a group means ______________________________.

2. Which is more, $\frac{3}{4}$ of 12 or $\frac{2}{3}$ of 15? ____________

Make a drawing to prove your answer.

3. Shade $\frac{2}{5}$ of ☺ ☺ ☺ ☺ ☺ ⬠ ⬠ ⬠ ⬠ ⬠.

4. What fraction of the set is pentagons? __________

⬠ □ ☺ □ ☺ ⬠ ⬠ ☺ ⬠

Explain how you found the answer.

5. If there are 14 boys in a class and 15 girls, what fraction of the class is boys? __________

About what part of the class is this? ______

6. Segment one is 40 cm. Segment two is 60 cm. Compare the following by placing <, > or = in the circle.

a) $\frac{1}{2}$ of the first ○ $\frac{1}{3}$ of the second.

b) $\frac{1}{4}$ of the first ○ $\frac{1}{4}$ of the second.

c) $\frac{1}{5}$ of the first ○ $\frac{1}{6}$ of the second.

d) $\frac{3}{4}$ of the first ○ $\frac{1}{5}$ of the second.

7. a) How many minutes are in $\frac{1}{2}$ hr.? ____ $\frac{1}{4}$ hr.? ____ $\frac{1}{3}$ hr.? ____

Use the circles as clocks to show this.

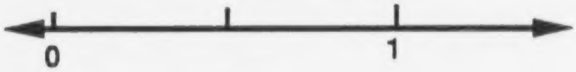
○ ○ ○

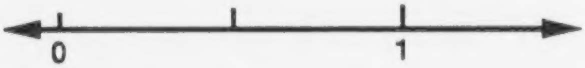
b) How many hours are in $\frac{1}{2}$ day? ____ $\frac{1}{4}$ day? ____ $\frac{1}{6}$ day? ____

c) How many meters are in $\frac{1}{2}$ km? ____ $\frac{1}{4}$ km? ____ $\frac{1}{8}$ km? ____

FRACTIONS ON A NUMBER LINE

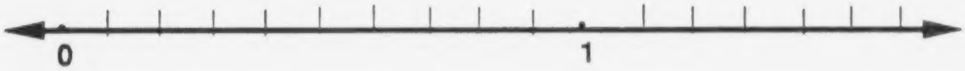
1. Show the location of each of the following on the number line:

$\frac{1}{2}$ of 1


$\frac{7}{8}$ of 1


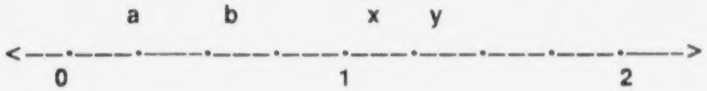
2. Mark the location of the following fractions on the number line.

$\frac{2}{5}$ $\frac{1}{4}$ $\frac{6}{5}$ $\frac{12}{10}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{3}{2}$

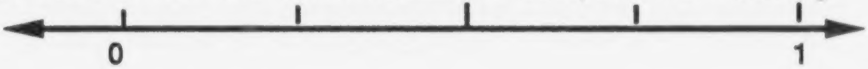


3. See the number line below. Give the number for

a ________ b ________ x ________ y ______



4. Use this number line when needed to compare the following:



a) $\frac{3}{5}$ is ☐ greater than ☐ less than ☐ equal to $\frac{1}{2}$.

b) $\frac{7}{14}$ is ☐ greater than ☐ less than ☐ equal to $\frac{1}{2}$.

c) $\frac{3}{8}$ is ☐ greater than ☐ less than ☐ equal to $\frac{1}{2}$.

d) $1\frac{1}{8}$ is ☐ greater than ☐ less than ☐ equal to 1.

e) $\frac{2}{3}$ is ☐ greater than ☐ less than ☐ equal to $\frac{3}{4}$.

5. Check the square to answer each of the following:
For each part, sketch a number line to show that your answer is correct.

a) Is $\frac{3}{8}$ closer ☐ to 0 or ☐ to $\frac{1}{2}$?

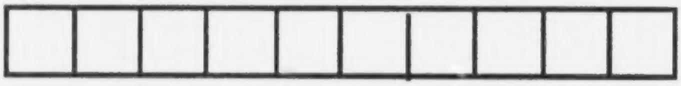
b) Is $\frac{1}{16}$ closer ☐ to 0 or ☐ to $\frac{1}{4}$?

c) Is $\frac{2}{3}$ closer ☐ to $\frac{1}{2}$ or ☐ to 1?

6. Shade $\frac{1}{10}$ of the figure.

Shade $\frac{3}{10}$ of the figure.

Shade $\frac{1}{5}$ of the figure.



Shade $\frac{2}{5}$ of the figure.

Use the shaded figures to compare $\frac{1}{5}$ and $\frac{1}{10}$; $\frac{2}{5}$ and $\frac{3}{10}$.
Place <, > or = in the square.

$\frac{1}{5}$ ☐ $\frac{1}{10}$ $\frac{2}{5}$ ☐ $\frac{3}{10}$

EQUIVALENT FRACTIONS

1. Shade $\frac{1}{5}$ of the figure.

Shade $\frac{2}{10}$ of the figure.

Use the shaded figures to compare $\frac{1}{5}$ and $\frac{2}{10}$.

Place <, > or = in the square. $\frac{1}{5}$ ☐ $\frac{2}{10}$

2. Make drawings to show that the following fractions are equivalent:

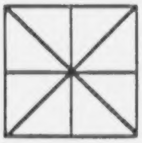
a) $\frac{1}{3} = \frac{2}{6}$

b) $\frac{3}{4} = \frac{6}{8}$

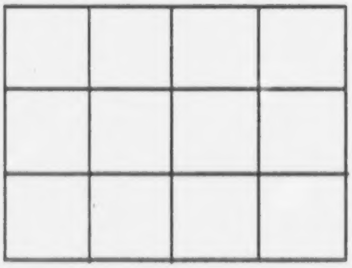
c) $\frac{1}{2} = \frac{5}{10}$

3. Every number has many names. Give as many as you can for these numbers using the figures given. Each figure represents one whole.

One = $\frac{2}{2}$ ______
= ______
= ______



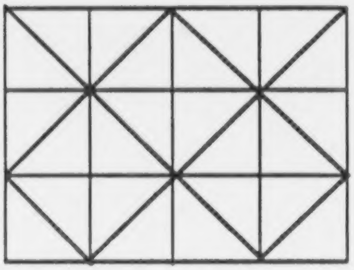
One half = ______
= ______
= ______



Two thirds = ________

= ________

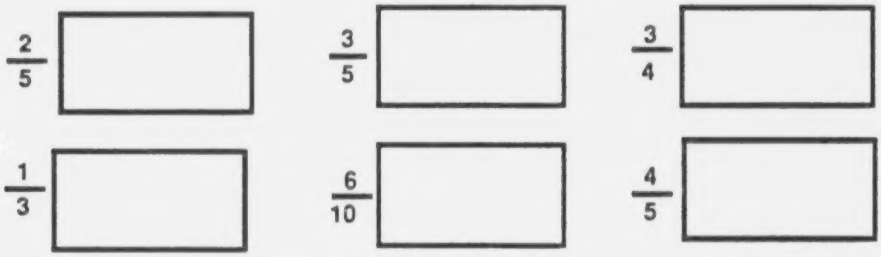
= ________



4. Select equal pairs and write the equality. For example, $\frac{1}{2} = \frac{5}{10}$.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{5}, \frac{1}{4}, \frac{6}{8}, \frac{6}{9}, \frac{5}{10}, \frac{2}{10}, \frac{4}{16}$

5. Shade the rectangles and compare the shaded areas to compare the numbers.



$\frac{2}{5}$ ☐ $\frac{1}{3}$ $\frac{3}{5}$ ☐ $\frac{6}{10}$ $\frac{3}{4}$ ☐ $\frac{4}{5}$

6. Match equal numbers by drawing a line to the proper point on the number line for each number.

$\frac{1}{4}$ of 2

$\frac{1}{2}$

half of a half

$\frac{10}{20}$

4 eighths

$\frac{1}{5}$

$\frac{1}{4}$ of $\frac{1}{4}$

$\frac{1}{10}$ of 2

$\frac{5}{10}$

$\frac{1}{4}$

two tenths

$\frac{6}{5}$

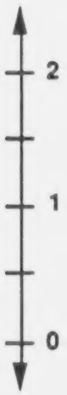
$1 - \frac{4}{5}$

$\frac{2}{8}$

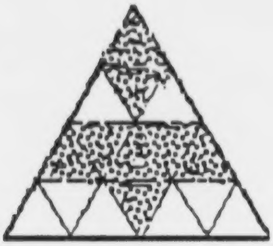
24 twentieths

$\frac{4}{20}$

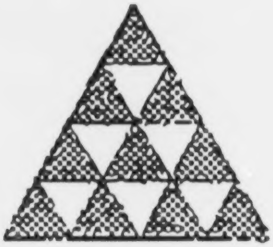
$\frac{5}{20}$



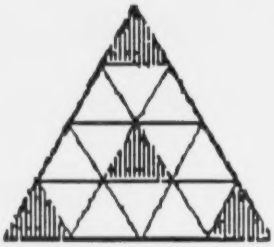
7. In each of the following, name the fractional part shaded.



a) ____________

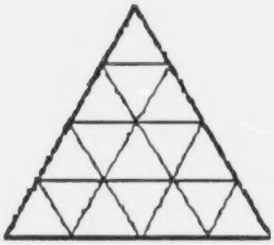


b) ____________

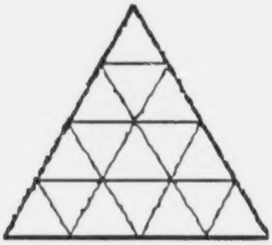


c) ____________

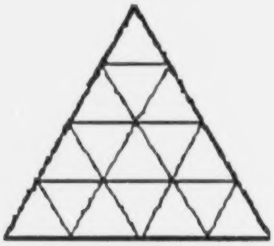
8. Shade the indicated fractional part of each of the following:



a) Shade $\frac{3}{4}$



b) Shade $\frac{3}{8}$



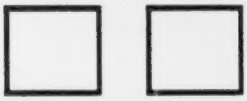
c) Shade $\frac{3}{16}$

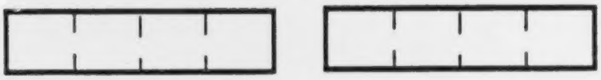
Explain how you decided the number of small triangles to shade in Part a).

MORE COMPARING FRACTIONS

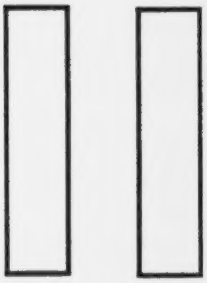
1. We can use a variety of units.

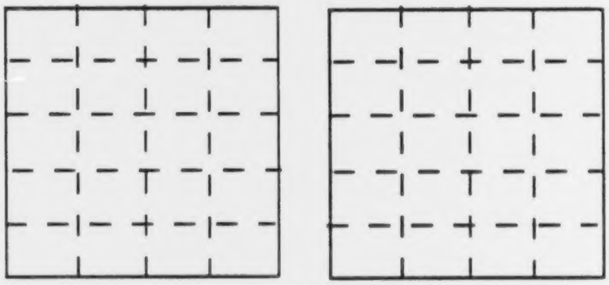
Shade $\frac{3}{4}$ and $\frac{4}{5}$ of each unit. Compare the fractions.



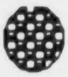


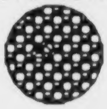
$\frac{3}{4}$ ☐ $\frac{4}{5}$ $\frac{3}{4}$ ☐ $\frac{4}{5}$





$\frac{3}{4}$ ☐ $\frac{4}{5}$ $\frac{3}{4}$ ☐ $\frac{4}{5}$

2. Would you prefer to have half of this pizza, 

or half of this one? 

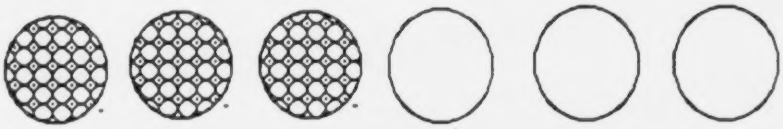
Explain why. ____________________

3. Arrange these fractions in **increasing** order: $\frac{7}{8}, \frac{1}{4}, \frac{1}{8}, \frac{1}{2}, \frac{3}{4}, \frac{8}{8}, \frac{3}{8}, \frac{5}{8}$.

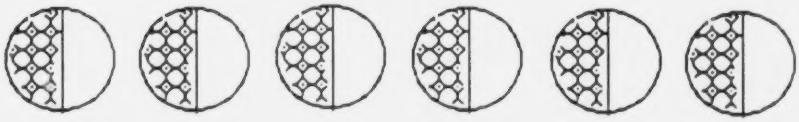
4. Arrange these fractions in **decreasing** order: $\frac{1}{9}, \frac{1}{3}, \frac{7}{9}, \frac{2}{3}, \frac{5}{9}, \frac{2}{9}, \frac{4}{9}$.

A FRACTION AS A QUOTIENT

1. To show 6 pizzas shared equally between 2 people, you could shade them in the following ways:



OR



Since division is used in sharing-equally situations, you could express this as $6 \div 2 = \frac{6}{2} = 3$

2. If **four** people share these 3 mini pizzas equally, shade the figures to show **two ways** you could show each person's share.

Give the fraction for the amount each would get. ______

This sharing equally situation can be expressed

$\_\_\_ \div \_\_\_ = \frac{3}{4}$

The **fraction** $\frac{3}{4}$ is the same as the **quotient** ___ ÷ ___.

3. If two bottles of cola are to be shared equally among three people, what part of a bottle will each get? ________

a) Make a drawing to show how you could find the answer.

b) What division expression would you use for this situation? ______

$$\frac{\bigcirc}{\square} = \bigcirc \div \square$$

4. If two people share **3** mini pizzas equally, how much will each get?

____

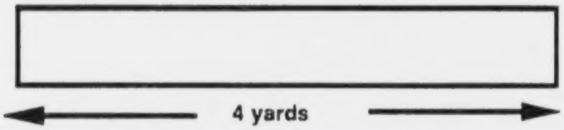
Shade the figures to show **two ways** you could share them.

Give the division expression, a fraction, and a mixed number for the amount each would get.

_______________ _______________ _______________

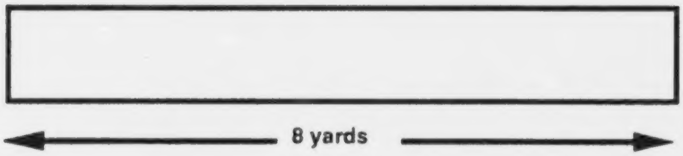
5. At a party you are planning, guests will be seated at small tables, with the food arranged on three larger tables.

a) If you are covering the smaller tables and have four yards of paper left for five tables, how much paper can you use for each?



4 ÷ 5 = ______

b) If you have 8 yards of the paper you want to use for the three food tables, how much paper can you use for each?



Show this as division: ○ ÷ □

as a fraction: ________

and as a mixed number: ________

ADDING AND SUBTRACTING FRACTIONS AND MIXED NUMBERS

1. Draw shaded figures to represent the following. Write each as a sum of a whole number plus a fraction.

$2\frac{1}{3}$ ______

$\frac{11}{8}$ ______

2. Draw shaded figures to represent the following. Use your drawings to find each sum.

$2\frac{1}{2} + 1\frac{1}{4}$ ______

$3 + \frac{1}{3}$ ______

3. Draw a square with side 6 cm. Divide it into 3 equal parts. Color in $\frac{2}{3}$ of the square. What portion is uncolored? ________

What equation could you write to show the amount that is uncolored?

4. Adding and subtracting fractions

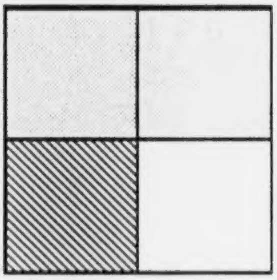
a) Show with a drawing what you need to add to $\frac{1}{4}$ to get 1.

so $\frac{1}{4} + \square = 1$ **or** $1 - \frac{1}{4} = \square$

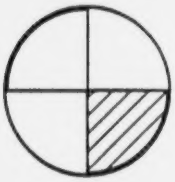
b) Show with a drawing what you need to add to $1\frac{3}{8}$ to get 3.

so $1\frac{3}{8} + \square = 3$ **or** $3 - 1\frac{3}{8} = \square$

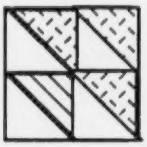
5. Complete the fractions to show different ways to make one whole.
The first one is an example.



$\frac{2}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}$ **= 1 whole**



$\frac{}{} + \frac{}{} + \frac{}{} = \frac{}{} =$ ________



$\frac{}{} + \frac{}{} + \frac{}{} = \frac{}{} =$ ________

6. A bicyclist has ridden $\frac{2}{3}$ of his trip. Is the part remaining more or less than the part ridden? __________
How do you know? (Explain and/or use a drawing)

7. Use three identical long rectangular strips. Fold one into **thirds**; fold one into **sixths**; fold one into **halves**. Tear **one third** from the **first**; tear **one sixth** from the **second**. **Compare** their **sum** with **one half** on the **third** strip. Explain your findings.

____

____

____

Write an equation to express the relationship.

8. How many different fractions with single-digit numerators and denominators can be formed using these cards? ______

$\boxed{2}\ \boxed{3}\ \boxed{5}\ \boxed{7}$

a) List them.

b) Can two equivalent fractions less than 1 be formed? __________
If not, which two are nearest in value? __________

c) Which fraction is the largest? __________
Which fraction is the smallest? __________

d) Which two fractions have the greatest sum? __________

Justify your answer. ______________________________

____

____

If you did not use a number line to help you answer this question, sketch one now and on it, locate the fractions you need to consider in answering the question. Explain how you could justify your answer using this number line.

Justify your answer using the number line. ______________

____

____

e) Which two fractions have the smallest sum? __________

Justify your answer. ______________________________

____

DECIMAL SENSE

FOCUS: Number Sense/Operation Sense

- Decimal numbers

PURPOSE: The students will . . .

- Understand the meaning of decimal numbers;
- Be able to compare decimal numbers based on understanding vs rules;
- Relate fractions to decimal numbers;
- Understand the algorithms for addition and subtraction based on the meanings of decimal numbers;
- Understand multiplication of decimals such as 1/10 of 1/10 = 1/100 using part-of-a-whole multiplication and number line models for decimals;
- Understand the meanings for time, money, lengths expressed with decimals; and
- Consider the reasonableness of real-world decimal numbers.

STUDENT BACKGROUND: Students need to have fraction sense on which to build decimal number sense. They should have an understanding of place value for whole numbers to extend to decimal numbers.

TEACHER BACKGROUND: Some understanding of decimal numbers is necessary for meaningful use of operations using them. Students need to understand relative magnitudes of numbers for estimating, for making sense of the outcomes of operations, and for comparing results (e.g., in "better buy" situations). Many students have misconceptions about decimals based on overgeneralizations of whole number concepts; they may believe that 0.89 > 0.9 because 89 > 9. Others think the larger the "number" following the decimal, the smaller the number (e.g., 1.95 < 1.8 because 95 > 8); others get the same results for this problem saying "hundredths are smaller than tenths."

MATERIALS: Base ten blocks (optional);
Part 1: PERCENT SENSE worksheet, calculators with constant function;
Part 2: SMALLER PARTS worksheet, calculators with constant function;
Part 3: COMPARING DECIMAL NUMBERS worksheet, digit cards for the games--Decimal Solitaire and In-a-Row Decimals;
Part 4: DECIMAL OPERATIONS worksheet, calculator for Target; and
Part 5: DECIMAL COMMON SENSE? worksheet.

LESSON DEVELOPMENT: If base ten blocks are available, use them in Parts 1 and 2 to introduce tenths, then hundredths, then thousandths. Students can be asked to use the blocks to build decimal numbers and to compare them--in place of the shading exercises in the worksheets or in addition to them. The meaning of decimals as fractions with denominators of powers of ten receives more attention in these exercises than the place value extension of whole numbers approach. Both approaches should be pointed out in class discussions of the exercises. The use of the calculator to check the counting exercises in Parts 1 and 2 may expose some student misconceptions which should be discussed as they arise.

Part 3: COMPARING DECIMAL NUMBERS

The number of decimal numbers between two decimals (**Problem 10**) will require discussion, with the extent of the discussion left to the teacher's discretion. The extended place value view of decimals could be discussed here, generalizing as appropriate for the particular students.

Instructions for two games that can be used to practice comparing decimals are included for use as the teacher deems appropriate:

Decimal Solitaire is a game for practicing comparison of decimals. It is played by individuals but can be adapted for play by pairs. Needed for each player are a set of digit cards 0-9, 2 cards with decimal points, one > card, and paper and pencil to record results. The student mixes the digit cards and places them face down in a pile, then draws 3 cards and uses them to make a decimal number with 2 decimal places. The student draws three more digit cards and makes as many numbers as possible less than the first number, recording the answers.

Students may be asked to devise a variation for play in pairs, or one can be suggested. For example, each student can form and record (without sharing it) a beginning number using the first three digit cards drawn. Students complete the game as above to see who could form more numbers smaller than the original number chosen.

In-A-Row-Decimals (Scott Resources, Inc.) is a game for group play. It requires a deck of 40 to 60 cards marked with decimal values such as 0.7, 0.69, 0.73, 0.046, 0.720, . . . The dealer deals 5 cards face up to each player. Cards are placed in a row as dealt and can not be rearranged. Remaining cards are placed face down to form a stack. Each player is trying to get five cards in order, smallest to largest from left to right. On each turn, a student draws a card from the stack and replaces any one of the cards in the row or discards it. The first player to get five cards in order wins.

Part 4: DECIMAL OPERATIONS

Some guidance is provided for the students in **Problem 9**, but more may be necessary. They can be told or left to discover that there is more than one answer for each answer in Part a). Students could discuss the reason there is more than one answer only for the addition problem, even though multiplication is also commutative.

Target is a game for two players that can be used with this lesson. Using a calculator, Player One enters any number. Player Two is to multiply this by another number so that the answer will be as near the target number, 100, as possible. Player One multiplies this answer by another number, attempting to get closer to 100. Play continues this way until the target 100.*** (* represent any digits following the decimal point) is reached on the calculator. Students should record the game steps and discuss the results. A variation is to use only division in reaching the target.

Some guidance is provided for the students in **Problem 9**, but more may be necessary. They can be told or left to discover that there is more than one answer for each answer in Part a). Students could discuss the reason there is more than one answer only for the addition problem, even though multiplication is also commutative.

Part 5: DECIMAL COMMON SENSE?

Since students may lack the background to judge the reasonableness of many of the numbers in Problems 3 and 4, they could be asked to predict and then to do research to verify or disprove their predictions. The class could be polled, with various factions doing research to prove their own predictions correct, or each student or group could be asked to research one area of dispute.

ANSWERS:

Part 1: PERCENT SENSE

1. 0.3, 0.2
2. Students are to use the constant function on the calculator to count by 0.1.
   0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0,1.1, 1.2, 1.3, 1.4
   0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0,1.1, 1.2, 1.3, 1.4
   10 tenths = 1; 5 tenths = one half; one half = 0.5
   one = 1 or 1.0

ANSWERS:

3. 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8
0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8

4. 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7
1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7
2.5

5. Shade 1 and 0.2 of second; shade 2 columns; shade 2 and 8 tenths of third; shade 1

6. 14/100 = 0.14; 57/100 = 0.57

7. 10/100 = 1/10
4, 1, 4

8. 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, 0.11, 0.12, 0.13, 0.14
Same
1 = 100 hundredths; one half = 50 hundredths, 2 1/2 = 250 hundredths
Less; 0.1

9. 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7
Same

10. 0.34 is 3 tenths which is 30 hundredths plus 4 hundredths.

11. 75/100; 0.75; 1/4

Part 2: SMALLER PARTS

1.

Fraction in lowest terms	Fraction with denominator 100	Decimal
$\frac{2}{5}$	40/100	0.4
9/25	$\frac{36}{100}$	0.36
1/20	5/100	0.05
3/10	30/100	0.30

2. 1000; 1/1000; 2/10 = 20/100 = 200/1000

3. Shading

4. a) 3 + 3 tenths + 3 hundredths + 3 thousandths
b) 4 tenths
c) 1 + 9 hundredths
d) 2 + 1 tenth + 4 thousandths
e) 1 + 5 thousandths
f) 5 tenths + 6 hundredths

5. a) different b) same c) different d) different
e) same f) same g) different

ANSWERS:

Part 3: COMPARING DECIMAL NUMBERS

1. $<$ $<$
If the whole number parts are the same, the larger has more tenths. If the tenths are the same, the larger has more hundredths. (If students suggest equalizing the number of decimal places and using the larger, encourage them to understand a more meaningful-based alternative method.)
2. $>$ $<$
3. See number 1. Go to the largest place value in which the two numbers differ; the larger in this place is the larger number.
4. 0.098, 0.2, 0.201, 0.30, 0.459, 0.46
5. 0.04, 0.233. 0.4, 0.406, 1.298, 1.30
6. a) 0 b) 0 c) 1 d) 1 e) equally close
7. Answers vary; approximate answers are
a) A-0.25 B-0.65 C-1.15 D-1.5 E-1.85 F-2.35 G-3.2
b) A-0.1 B-0.2 C-0.5 D-0.75 E-0.95 F-1.04 G-1.21
9. 0.5, 0.25, 0.6, 0.1, 0.75, 0.2
10. Answers vary. Infinitely many in each case.

Part 4: DECIMAL OPERATIONS

1. 0.8; $0.3 + 0.5 = 0.8$
2. 0.5; $0.9 - 0.4 = 0.5$
3. 0.45; $0.3 + 0.15 = 0.45$
4. 0.01; $0.41 - 0.4 = 0.01$
5. 0.01; 0.01; $0.1 \times 0.1 = 0.01$
Less; one tenth of one tenth is one of ten equal parts of one of ten equal parts of 1, so it is one of 100 equal parts of 1.
6. 0.001; 0.001; $0.1 \times 0.01 = 0.001$
less; less;
One tenth of one hundredth is not a whole hundredth, so is less.
7. 50; b
8. 500; $0.5 \div 0.001 = 500$
9. Part a) has more than one possible answer for each part.
a) Largest: $98.2 + 7.5 = 105.7$ Smallest: $25.9 + 7.8 = 33.7$
b) Largest: $98.65 - 10.3 = 88.35$ Smallest: $10.35 - 9.86 = 0.49$
c) Largest: $95.2 \times 8.7 = 828.24$ Smallest: $27.9 \times 5.8 = 161.82$
d) Largest: $986.4 \div 1.2 = 822$ Smallest: $124.6 \div 9.8 = 12.7$

ANSWERS:

Part 5: DECIMAL COMMON SENSE?

1. b) $4.08 c) $10.00 d) $.07
2. b) $28.40 c) $.45 d) $2.02 e) $.71 f) $.30
3. a) 238.5 b) 20.5 x 15.5 c) $19514.00 d) $315.00, 70.2%
   e) 77% f) 16.0 g) 9.17 to 9.88
4. a) 750 b) 360, 440 c) 1200 d) 20, 307000, 14000
   e) 100.9, 60.5, 0.41 f) 1.9 to 2.4 g) 0.085-0.09 oz.
5. a) False, 1 hour and 24 minutes b) False, 23 pounds and 8 ounces
   c) 2 dollars and 10 cents d) True
6. a) 15 b) 4 c) 1 1/2 feet or 18 inches or 1 foot 6 inches d) 4

SOURCE: Adapted from

Markovits, Z. & Sowder, J. (1989). *Understanding decimals: Instructional materials: Middle grades.* NSF Grant No. NDR-8751373. Center for Research in Science and Mathematics Education, San Diego State University, San Diego.

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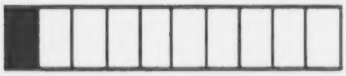
DECIMAL SENSE

1. Tenths



$\frac{1}{10}$ is shaded.
Written as a decimal, this is **0.1.**

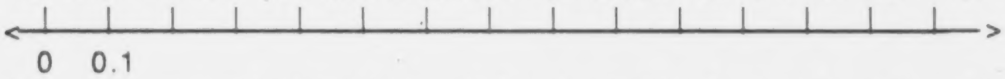
Shade three tenths.
The decimal form is ____.

Shade one fifth.
Write the decimal form for one fifth. ___

2. Count by one-tenths from zero:

0, 0.1, __, __, __, __, __, __, __, __, __, __, __, __, __

For each number you counted, label its location on the number line.



Now use your calculator to count by one-tenths from zero and record each number you get each time you press "=" on the line below "=":

0 + 0.1 = = = = = = = = = = = = =

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Did you get the same numbers both ways? Explain any differences.

How many tenths make 1? ___ one half? ____ Write one half in decimal form. _____

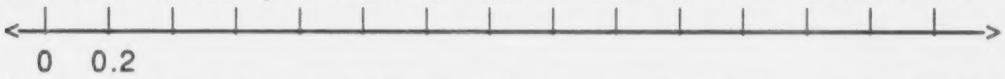
How many tenths make 2? ___ $2\frac{1}{2}$ ___. Write $2\frac{1}{2}$ in decimal form. ___

Write the number one as a decimal number. ____

3. Count by two-tenths from zero:

0, 0.2, __, __, __, __, __, __, __, __, __, __, __, __, __

For each number you counted, label its location on the number line.



Now use your calculator to count by two-tenths from zero and record each number you get each time you press "=" on the line below "=":

0 + 0.2 = = = = = = = = = = = = =

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Did you get the same numbers both ways? Explain any differences.

4. Count by five-tenths from zero:

0, 0.5, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___

For each number you counted, label its location on the number line.

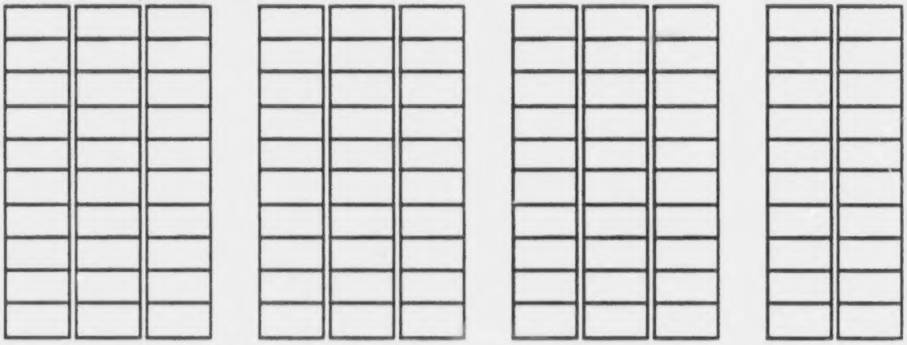
Now use your calculator to count by five-tenths from zero and record each number you get each time you press "=" on the line below "=":

0 + 0.5 = = = = = = = = = = = = =

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Did you get the same numbers both ways? Explain any differences.
From your counting, how much is 5 x 0.5? _____
Confirm your answer using a calculator.

5. Each column represents 1.



Shade to show 1.2 2 2.8 1.0

6. Hundredths

Since 0.1 is one tenth, what is 0.11? We found that it is **not** eleven tenths when we counted by one-tenths on the calculator.

0.11 = 0.1 + 0.01, and 0.01 is one-hundredth:

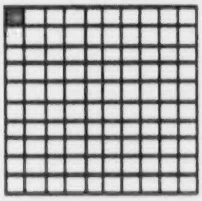
So, $0.01 = \frac{1}{100}$; $0.1 = \frac{\_\_\_}{100}$, so $0.11 = \frac{\_\_\_}{100}$ (Give the missing numerators).

Write 0.11 in words:

Explain what 0.11 means.

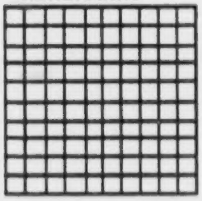
Shade to show the parts below and give the fraction and decimal forms.

One hundredth



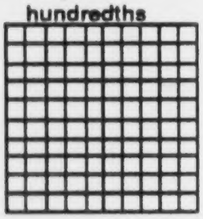
$\frac{1}{100} = 0.01$

Fourteen hundredths



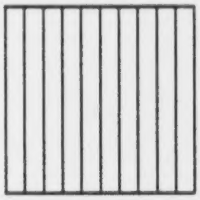
___ = ___

Fifty-seven hundredths

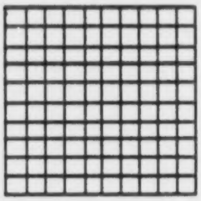


___ = ___

7.



Shade $\frac{1}{10}$



Shade $\frac{10}{100}$

Complete the following:

Each $\frac{10}{100} = \frac{\_\_}{10}$.

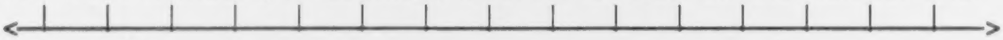
$\frac{14}{100} = \frac{10}{100} + \frac{\_\_}{100} = \frac{\_\_}{10} + \frac{\_\_}{100}$

Write these equations in decimal form.

8. Count by one-hundredths from zero:

0, 0.01, __, __, __, __, __, __, __, __, __, __, __, __, __

For each number you counted, label its location on the number line.



0 0.01

Now use your calculator to count by one-hundredths from zero and record each number you get each time you press "=" on the line below "=":

0 + 0.01 = = = = = = = = = = = = =

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Did you get the same numbers both ways? Explain any differences.

How many hundredths would make 1? ___ one-half? ____ $2\frac{1}{2}$ ___?

Would 10 x 0.01 be more than 10 or less than 10? ____

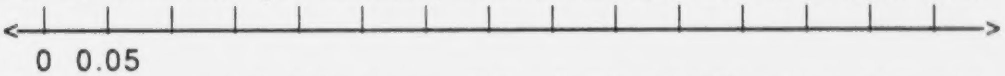
From your counting, how much is 10 x 0.01?________

Confirm your answer using a calculator.

9. Count by five-hundredths from zero:

0, 0.05, __, __, __, __, __, __, __, __, __, __, __, __, __

For each number you counted, label its location on the number line.



Now use your calculator to count by five-hundredths from zero and record each number you get each time you press "=" on the line below "=":

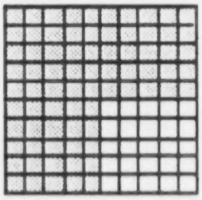
0 + 0.05 = = = = = = = = = = = = =

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Did you get the same numbers both ways? Explain any differences.

10. Explain in words why 0.34 is thirty-four hundredths. Make a drawing to support your answer.

11.



How many hundredths are shaded?

Write the fraction: $\frac{\square}{100}$

Write the decimal: ☐

How many fourths are shaded?

Write the fraction: $\frac{\square}{4}$

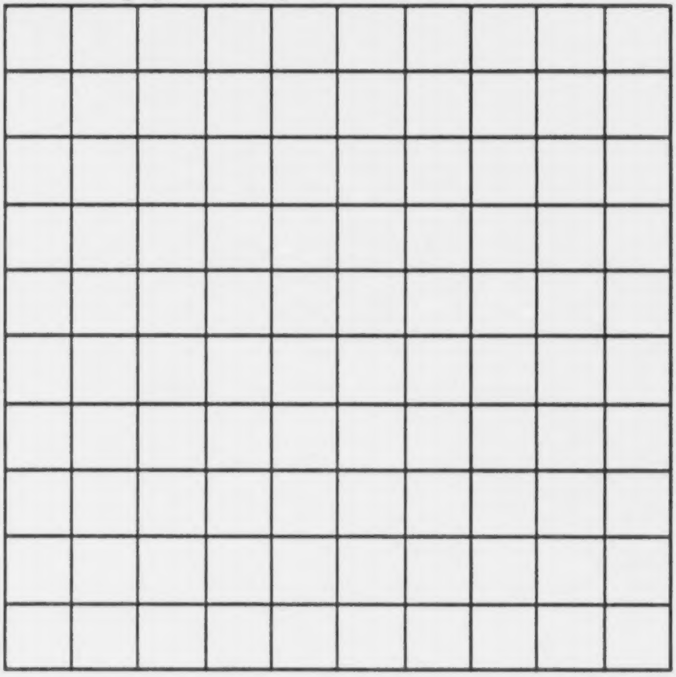
So $\frac{\square}{4} = \frac{\square}{100} = \boxed{}$ (decimal)

SMALLER PARTS

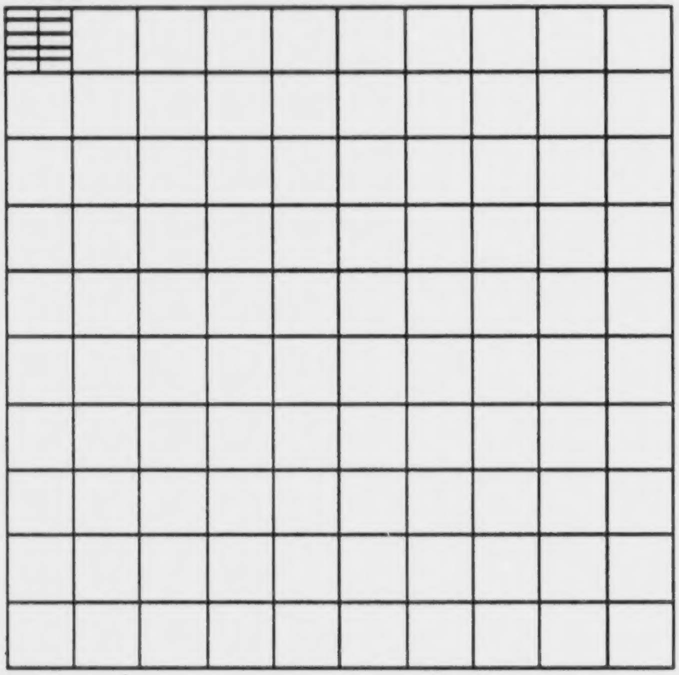
1. a) Complete the following table:

Fraction in lowest terms	Fraction with denominator 100	Decimal
$\frac{2}{5}$		
	$\frac{36}{100}$	
		0.05
		0.30

b) For one of your answers in Part a), shade this grid and explain how the shading justifies your answer.



2. For the first column in the grid below, continue to divide each square (representing $\frac{1}{100}$) into 10 equal parts.



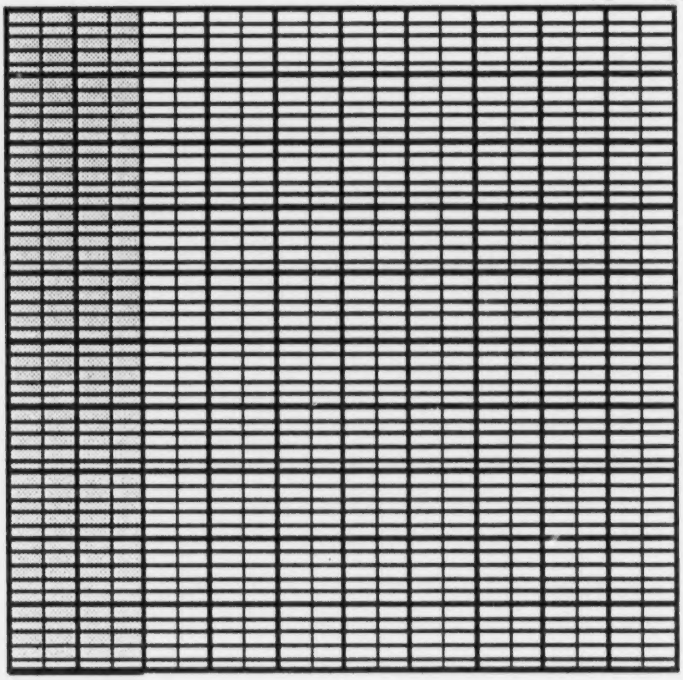
If we divide each $\frac{1}{100}$ of the grid into 10 equal parts, how many parts will there be in the whole grid? ______

Each of these parts will be what fraction of the whole? ________

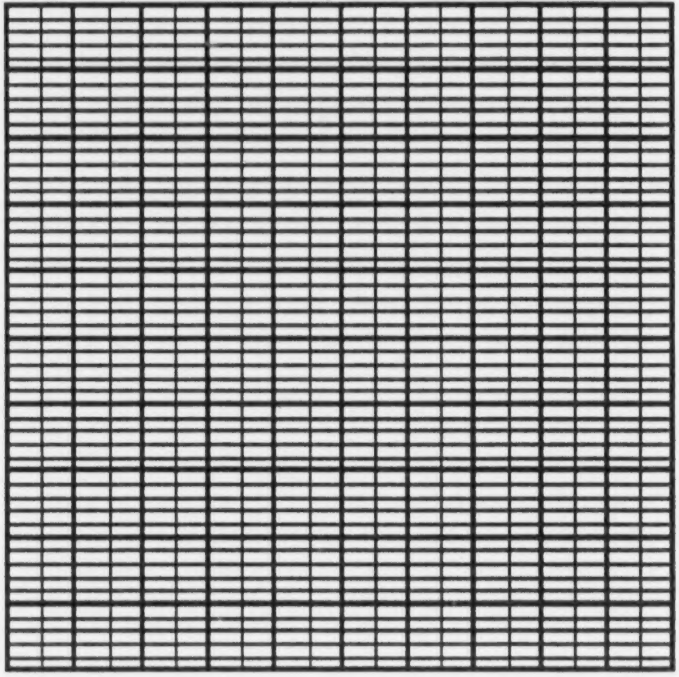
In decimal form, we write 0.001 for this number.

Explain the relationships among one tenth, one hundredth, and one thousandth. How is this similar to the way the "places" in whole numbers are related? How is it different? Explain in writing.

What part is shaded in the following? ______ $\frac{\quad}{10} = \frac{\quad}{100} = \frac{\quad}{1000}$



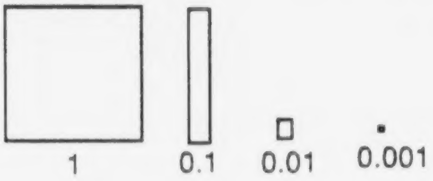
3. Shade one hundred twenty-four thousandths of this:



One hundred twenty-four thousandths

$$\begin{aligned}
&= \frac{100 + 20 + 4}{1000} \\
&= \frac{100}{1000} + \frac{20}{1000} + \frac{4}{1000} \\
&= \frac{1}{10} + \frac{2}{100} + \frac{4}{1000} \\
&= 0.1 + 0.02 + 0.004 \\
&= 0.124
\end{aligned}$$

4. Make a drawing to represent each of the numbers below using sizes similar to the following. (The shapes representing the parts are not exactly to scale. Your drawings do not have to be the exact sizes of these pieces.)



a) 3.333

b) 0.4

c) 1.09

d) 2.104

e) 1.005

f) 0.56

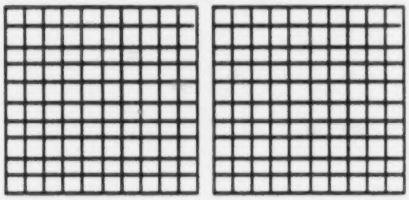
5. On the line, write "same" if the two names given represent the same number. Write "different" if they do not.

a)	1.01	and	1.1	__________
b)	1.1	and	1.10	__________
c)	0.008	and	0.800	__________
d)	10.01	and	10.10	__________
e)	4.9	and	4.90	__________
f)	1	and	1.00	__________
g)	0.006	and	0.060	__________

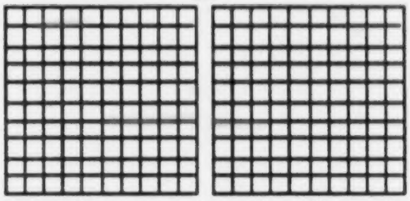
COMPARING DECIMAL NUMBERS

1. For each pair below, tell if the first is <, =, or > the second. Shade the pairs of figures to show that the comparison is correct.

0.84 ☐ 0.9



0.24 ☐ 0.33

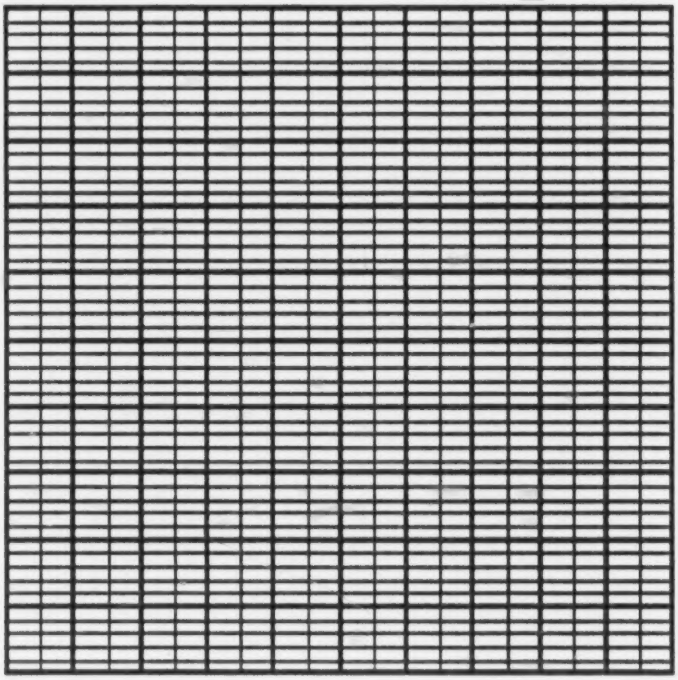


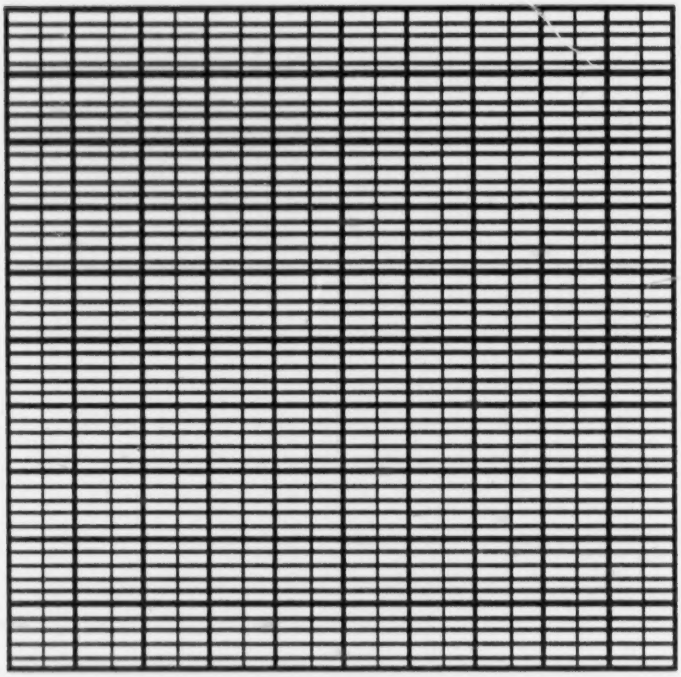
Explain how to compare decimals that have tenths and hundredths places given:

2. For each pair below, tell if the first is <, =, or > the second. For one of the problems, shade the figures to show that the comparison is correct.

0.8 ☐ 0.792

0.234 ☐ 0.24





3. Explain how to compare any two decimals.

4. Arrange the following from smallest to largest:

0. 2   0.098   0.30   0.46   0.459   0.201

_______ _______ _______ _______ _______ _______

5. Arrange the following from smallest to largest:

0. 233   1.298   1.30   0.406   0.4   0.04

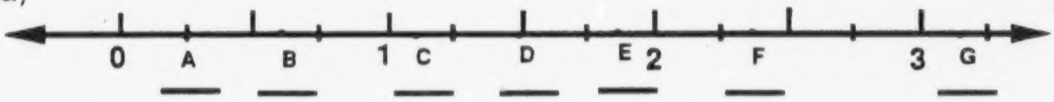
_______ _______ _______ _______ _______ _______

6. Use drawings when you need them to answer the following. Write "equally close" if the number is the same distance from both choices.

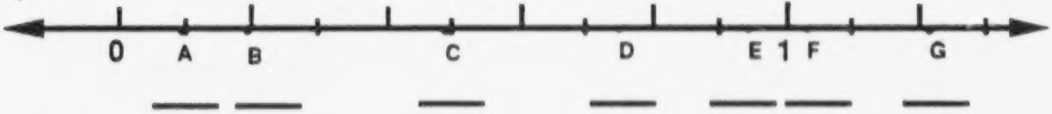
a) Is 0.4 closer to 0 or to 1? _____
b) Is 0.199 closer to 0 or to 1? _____
c) Is 0.51 closer to 0 or to 1? _____
d) Is 1.3 closer to 1 or to 2? _____
e) Is 1.5 closer to 1 or to 2? _____

7. Give approximate names for the points indicated on the number line:

a)

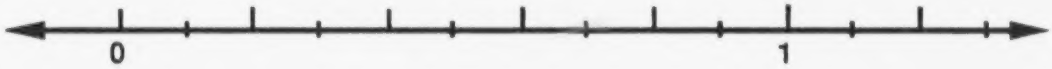


b)

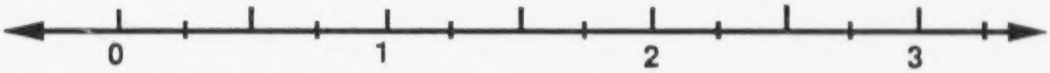


8. Use points to show the approximate locations of the following on the number line. Label each point.

a) 0.43 0.78 0.3 1.20 1.005 0.085 0.005



b) 2.36 1.5 3.11 0.75 1.05 0.1 2.0



9. For each number in the first row, find one in the second row that is the same value.

$\frac{1}{2}$ = ___ $\frac{1}{4}$ = ___ $\frac{3}{5}$ = ___ $\frac{1}{10}$ = ___ $\frac{3}{4}$ = ___ $\frac{1}{5}$ = ___

0.10 0.6 0.5 0.75 0.2 0.25

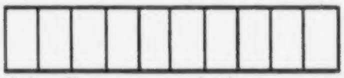
10. Decimals between decimals. Draw number lines to show the locations of the points in the following.

a) Can you name any decimal numbers between 0.4 and 0.5? ___
If so, give one. _____ How many are there? ___________

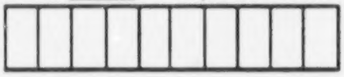
b) Can you name any decimal numbers between 0.14 and 0.15? ___
If so, give one. _____ How many are there? ___________

DECIMAL OPERATIONS

1. Shade 0.3 of the figure then shade 0.5 more. How much is shaded all together? ____ Express this in a number sentence. ________________

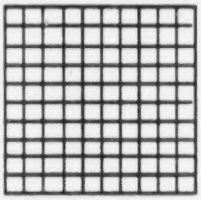


2. If you shade 0.9 of this figure and then erase the shading from 0.4, how much remains shaded? ____ Express this in a number sentence.



____

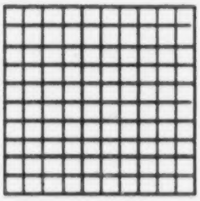
3. Shade 0.3 of the figure then shade 0.15 more. How much is shaded all together? ____ Express this in a number sentence. ________________



Explain why the answer is **not** 0.18.

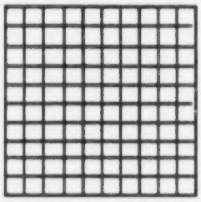
4. If you shade 0.41 of this figure and then erase the shading from 0.4, how much remains shaded? ____ Express this in a number sentence.

____



Explain why the answer is **not** 0.37.

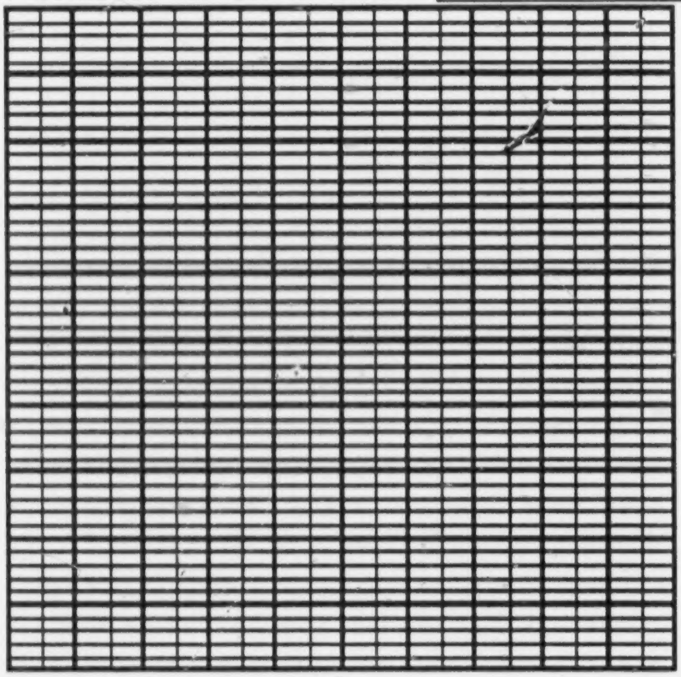
5. How much of the following figure is shaded if you shade 0.1 of 0.1 of it? ___ Use your calculator to multiply 0.1 x 0.1. What is the result? __ Write a number sentence for 0.1 of 0.1. __________________________



Is your answer more or less than 0.1? ____ Explain the reason your answer is correct.

6. How much of the following figure is shaded if you shade 0.1 of 0.01 of it? ___ Use your calculator to multiply 0.1 x 0.01. What is the result? __

Write a number sentence for 0.1 of 0.01. ____________________



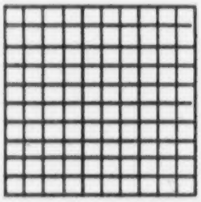
Is your answer more or less than 0.1? ______

Is your answer more or less than 0.01? ________

Why is this true?

7. How many hundredths are in five tenths? ☐ Use the figure below to help decide. Which number sentence below expresses this relationship?______

a) 0.01 ÷ 0.5 = ☐ b) 0.5 ÷ 0.01 = ☐



8. How many thousandths are in five tenths? _____ (Use the figure in Problem 6 if you need it.) Write the number sentence that expresses this relationship. ____________________

9. Place each of the given digits just once in the problem indicated to give the largest possible answer, then to give the smallest possible positive answer. Give the answers.
Discuss your answers with your classmates, and use your calculator to check if there are differences of opinion. When you agree on the correct answers, try to decide why these are the places that "work" in each case. Test your theory using some different numbers.

a) Use 2, 5, 7, 8, and 9 in this problem.

Largest __ __ . __ + __ . __ = ______________

Smallest __ __ . __ + __ . __ = ______________

Explain your reasoning.

b) Use 3, 1, 0, 5, 6, 8, 9 in this problem. (Do not use zero as the first digit in a number larger than 1.)

Largest __ __. __ __ - __ . __ __ = ______________

Smallest __ __. __ __ - __ . __ __ = ______________

Explain your reasoning.

c) Use 2, 5, 7, 8, and 9 in this problem. (There are some very close choices for this one. Predict which way will give the biggest answer, but try more than one way on your calculator. Consider carefully where the 5 and 7 should go for the largest product. Think about the reasons for the differences in the answers.)

Largest __ __ . __ x __ . __ = ______________

Smallest __ __ . __ x __ . __ = ______________

d) Use 1, 2, 4, 6, 8, 9 in this problem (This is much easier than Part c.)

Largest __ __ __ . __ ÷ __ . __ = ______________

Smallest __ __ __ . __ ÷ __ . __ = ______________

DECIMAL COMMON SENSE?

1. Rewrite the following using a dollar sign and decimals:

a) 4 dollars and 30 cents	$4.30
b) 4 dollars and 8 cents	_____
c) 10 dollars	_____
d) 7 cents	_____

2. Write the following calculator answers in dollars and cents

a) 4.56	$4.56
b) 28.4	_____
c) 0.45	_____
d) 2.02	_____
e) 0.709	_____
f) 0.3	_____

3. What's wrong with these decimals? Rewrite any decimals that don't make sense; place the decimal point correctly. Make your best guess if you are unsure; then do some research to check your answers.

a) We drove 2385 miles on the freeway during the last 4 hours.

____

b) My living room is 205 feet long and 155 feet wide. _____________

c) The 1988 per capita (per person for the year) income in Alaska was $1951.40. ___________

d) The 1988 average weekly earnings for women with full-time jobs was $3150.00. This was 7.02% of the average for men's weekly earnings that year. _________

e) The world's literacy rate (percent who can read) in 1980 was 7.7% for men and 66% for women. _________

f) The total land area of the Earth is 160 times the land area of the United States. _______

g) A volleyball weighs 91.7 to 98.8 ounces. _____________

4. Place the decimal point in the numbers in the following so the numbers are reasonable. **You may need to add zeros before or following the digits given. You may also need to do some research.**

a) The Wright brothers' Flyer I, first flown in 1903, weighed 75 pounds.

b) Females use about 36 Calories per hour walking uphill, and males use about 44.

c) An adult male polar bear can weigh up to 12 pounds.

d) The world's heaviest sea creature is a little more than 2 times as heavy as the heaviest land creature. The blue whale weighs about 307 pounds and the African elephant weighs about 140 pounds.

e) Nolan Ryan threw his fastest recorded pitch on August 20, 1974. The ball, traveling at 1009 miles per hour, traveled the 605 feet from the pitcher's mound to home plate in 41 second(s).

f) San Diego County's population grew from 19 million people to 24 million between 1980 and 1988.

g) A table tennis ball weighs 85-90 ounces.

5. Are these statements true or false? For each that is false, give the correct information. If you are unsure, draw pictures to help decide.

__ a) 1.4 hours = 1 hour and 4 minutes
__ b) 23.5 pounds = 23 pounds and 5 ounces
__ c) $2.1 = two dollars and one cent
__ d) 1.2 meters = 1 meter and 2 centimeters

6. Complete each of the following:

a) 1.25 hours = 1 hour and ____ minutes
b) 14.25 pounds = 14 pounds and ____ ounces
c) 1.5 yards = 1 yard and ______
d) 2.33 feet is about 2 feet and ___ inches

PERCENT SENSE

FOCUS: Meaning-Centered Lesson
- Percent
- Proportions

PURPOSE: The student will . . .
- Learn the meaning of percent using various models of the whole;
- Apply percent concepts using models to find the percent, the part, or the whole intuitively;
- Use estimation and mental computation with percents in the abstract and contextual situations; and
- Learn to find percents, parts, and wholes using models to develop proportional thinking.

STUDENT BACKGROUND: The students need to have number sense for fractions and decimals. Some vocabulary from geometry is used (midpoint), and other geometry vocabulary would facilitate the discussion of some exercises.

MATERIALS: It is suggested that manipulatives be used and that transparencies of the exercises be made to use in frequent class discussion during these lessons.
Part 1: PERCENT SENSE worksheet; paper for squares for folding, colored pencils (optional)
Part 2: USE YOUR PERCENT SENSE worksheet
Part 3: BENCHMARKS FOR ESTIMATING worksheet, meter stick
Part 4: MENTAL COMPUTATION WITH PERCENTS worksheet
Part 5: PROPORTION SENSE worksheet

LESSON DEVELOPMENT: Many challenging exercises are included that may require teacher-led discussion. Much of the work is appropriate for group work, with student presentation of results (possibly on overhead transparencies) and class discussion of the results at frequent intervals suggested. The unit consists of five parts.

Part 1: PERCENT SENSE
This is an introduction to the concept of percent, including percents less than 1% and greater than 100%.

Part 2: USE YOUR PERCENT SENSE

In Part 2, students use the meaning of percent and fractional equivalents to find relationships among parts, especially to find percents larger than 100%.

In Problem 5, stress that percent depends upon defining the reference (100%) quantity. Drawing dotted lines may help.
Extension: Assign various pieces of the Tangram puzzle as the 100% quantity, and relate the other pieces to the 100% piece.

Part 3: BENCHMARKS FOR ESTIMATING

Part 3, Benchmarks for Estimating, encourages the use of benchmark percents and their fractional equivalents with number line models for estimating and using mental computation. Allow for the degree of latitude in your students' estimates you deem appropriate based on their estimation experience and ability.

Part 4: MENTAL COMPUTATION WITH PERCENTS

Have students share their methods of doing mental computation and comment that there are several equally efficient ways. Highlight advantages of especially efficient ways when they are offered; occasionally suggest an additional useful alternative that does not emerge spontaneously.

Part 5: PROPORTION SENSE

Part 5 aims to develop understanding of the relationships involved in using ratio and proportion to solve percent problems. The class might do the first problem as a group, with students suggesting how to divide the 100 grid into fifths to get three-fifths and counting the number of parts out of 100 in three-fifths. Students could then do the other problems with results discussed in class.

ANSWERS: Many answers in these exercises are approximations; students' answers may vary; accept reasonable estimates where estimates are requested.

Part 1: PERCENT SENSE

1. 50%, 24%, 40%
    0.5, 0.24, 0.4
    1/2, 6/25, 2/5

d) 25% is 1/4; 75% is 3/4.

e) 30% 10%
    40% 20%
    Sum = 100%

f) The parts in the center circle add up to more than 100% so are not sensible.

2. 65% not shaded
3. a) 50%, 25%
   b) 50%, 25%
   c) 75%, 50%
4. Grid 1 - half of 1 small square shaded; a) 200
   b) Grid 2 - half of entire figure shaded.
   c) One-half percent is half of 1/100 of the whole; one-half is 50% of the whole.
   d) Grid 2 - 50% shaded
   e) $\frac{1}{2}\% = \frac{1}{200}$; $\frac{1}{2}\% = 0.005$
5. 23 1/2 squares shaded; 12 3/4 squares shaded; 63 3/4 squares not.
6. Grid 2 - 65% Grid 3 - 150%
7. 60, 80, 100, 120, 150, 200
8. Answers vary.

Part 2: USE YOUR PERCENT SENSE

1. 4% - 4 pennies, $0.04
   55% - 50¢ and a nickel, $0.55
   142% - 50¢ + 50¢ (or $1), quarter, dime, nickel, 2 pennies; $1.42
   98% - 50¢, quarter, 2 dimes, 3 pennies; $0.98

Answers vary

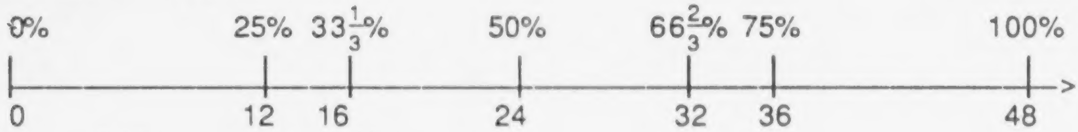
2. a) 9
   b) ≈ 4
   c) 32
   d) 25%, ≈30%
3 a) 1/4, 1/2, 3/4 the original rectangle
   b) One part, 3 parts
   c) One rectangle, 4 rectangles
4. a) Shade 1/3
   b) 1/4, 1/2, 3/4 of the square
   c) 2 parts
   d) 20% is one of the 6 congruent triangles formed by drawing the diagonals. 100% is 5 of these triangles.
   e) 100% is one of the small hexagons.
   f) 10% is one circle, one of the 16 equal parts, so 100% is ten circles.
   g) Sixteen congruent triangles are formed when the diagonals are drawn and the opposite midpoints of the sides are joined. Ten of these form the smaller figure.
5. a) < b) > c) = d) < e) = f) <

6.	a)	25	400
	b)	33 1/3	300
	c)	100	100
	d)	200	50

Part 3: BENCHMARKS FOR ESTIMATING

Throughout this part, any reasonable answer should be accepted, with emphasis on the fact that there is not just one correct answer. Reasonableness is to be determined by the teacher, based on the experience of the students.

1. 1/2, 1/4, 1/3, 3/4, 2/3, 1/8, 1/5, 1/10
3. a) 1/2, 60%   b) 0, 8%   c) 0, 0.006
4. 37.5%
5. 48   24



2/3   5 ≈ 12%

6. a) 10   b) 33 1/3%   c) 26   d) 60   e) 32   f) 50%
   g) 100   h) 30%   i) 10%   j) 30%   k) 4   l) 30
7. Answers vary.
8. A 25   B 16   C 14   D 7   E 40
   Sum = 102%; Close to reasonable
   A 1/4   B 1/6   C 1/8   D 1/10   E 1/3
   Sum should be 1. Sum = 0.97
10. b) B - 100% of B, C - 200% of B, D - 300% of B, A - 400% of B
    c) B - 100% of B, C - 200% of B, D - 200% of B, A - 300% of B
    d) A - 100% of A, B - 133% of A, C - 150% of A
    e) A - 100% of A, B - 250% of A, C - 300% of A, D - 550%

Part 4: MENTAL COMPUTATION WITH PERCENTS

These are possible methods. Encourage sharing of alternatives. Emphasize that there is more than one way to do mental computation, using the structure of the number system.

1. 5 Estimate using 10% + 1/2 of 10% ; double tax = estimate for tip
2. 4/5   6 left
3. Possible methods. Discuss the ones the students used. Emphasize that there is more than one way.
   a) \$50 - \$12   b) \$1.70 + 85¢ then \$17 - \$2.50
   c) 1/4 left to pay; \$4.75 (\$4 + 50¢ + 25¢)
   d) 10% ≈ \$11, so \$44.
4. 1/3 of \$48 is \$16; \$49 - \$16 > \$30 so have to wait and hope.

5. $50 discount is 33 1/3% of $150
6. 60% is $42, so 10% is $7, so 100% is $70.
7. Answers vary
8. Answers vary
9. 100

Part 5: PROPORTION SENSE worksheet

1. a) 10 b) 20 c) 50 d) 2 e) 50% f) 50%
2. 25, 25

Possible solution methods:

3. Shade 2 1/2 rows of the second grid = 20 squares + 10 half-squares = 25 our of 100.

or using fractions sense vs equal areas--5 is 1/4 of 20; 1/4 of the 100 grid is 25 squares.

4. Shade 1/5 of the columns of squares in the 100 grid = 20 squares
5. Shade 1/3 + 1/3 of 1/3 of the small squares in the 100 grid. Count the squares. About 44.
6. 75% (7 1/2 rows of the 100 grid) is 3/4. Shade three of the four rows from the 20, 15 of 20.
7. 40 out of 100 is 4 tenths or 2 fifths; 2/5 of 30 is 12; 12 out of 30.
8. 15% is 1 1/2 columns of the 100 grid. Since the 40 grid also has 10 columns, 15% of 40 is 1 1/2 columns of the 40 grid or 6.
9. Divide the empty square in half one direction and in sevenths the other -- 14 parts, so 7 out of 15 is the same as 50 out of 100.
10. 1 1/2 columns of 100 grid makes 30 so 3 columns make 60. There are 3 sixties and 1 column, equivalent to 2/3 of 30 so 20. So 30 out of 200 is the same as 15 out of 100.
11. 1% of 600 is 60, so 1/2% is 30.
12. More. 1 1/2 100s grids is 150% so 1 1/2 40s grids is 150% or 60.

SOURCE: Adapted from

Allinger, G. D. (1990, April). *Errors in learning percent: Diagnosis and remediation*. Presentation at NCTM Annual Meeting, Salt Lake City.

Mathematics Resource Project. (1977). *Ratio, proportion, and scaling*. Palo Alto, CA: Creative Publications.

Sobel, M. A. & Maletsky, E. M. (1988). *Teaching mathematics: A sourcebook of aids, activities, and strategies*. Englewood Cliffs, NJ: Prentice Hall.

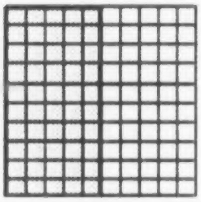
PERCENT SENSE

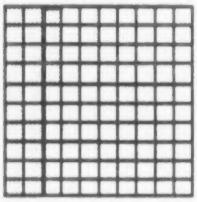
Percents are often used to convey important information. Of course, if the newspaper reader or television viewer does not understand percents, they may convey misinformation instead.

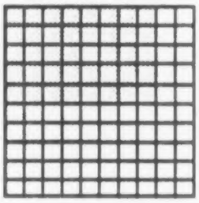
> By the year 2000, 20% of all Earth's animal species could be lost forever.
>
> You should leave about a 15% tip for a restaurant meal.
>
> Ninety-seven percent of the Earth's water supply is contained in our oceans, and 2% is frozen. We get our water from the 1% that is left, from Earth's surface and from groundwater.
>
> Forty percent of the pure water you use in your house is flushed down the toilet.

Percent means "per hundred" or "in each hundred." If you asked 100 students in this school if they watch the news, how many would say that they do? _____ Then ____% (same number) of those asked watch the news. If all of them say yes, then 100% of the students asked watch the news. 100% means "the whole thing."
If we divide the whole thing into 100 equal parts, each part is **1%**.
You know that 1 of 100 equal parts of the whole can also be called $\frac{1}{100}$ or **0.01**.

1. In the following, what percent of each grid is shaded?







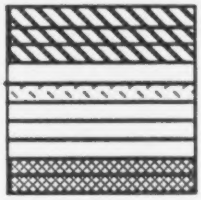
a) ____________ b) ____________ c) ______________

Express each of these percents as a decimal.
a) ____________ b) ____________ c) ______________

Express each of these percents as a fraction in simplest terms.
a) ____________ b) ____________ c) ______________

d) What fraction is 25%? _____ 75%? ______

e)

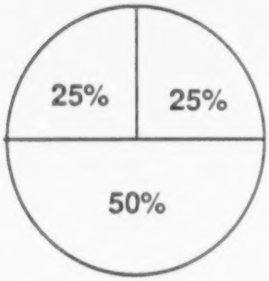


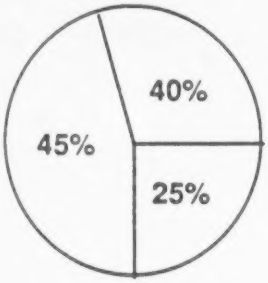
___ % is shaded ▨   ___ % is shaded ⬚

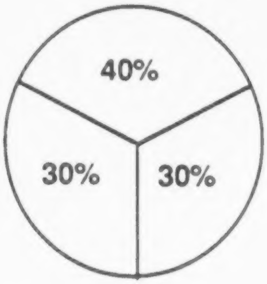
___ % is shaded □   ___ % is shaded ▩

What is the sum of these percents? ______

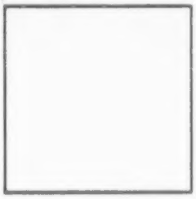
f) What's wrong with this picture, if anything? Are all of these percents sensible?
Explain your answer.

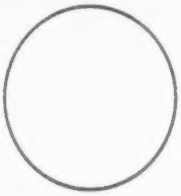






2. Shade each of the following to show about 35% shaded.
What percent of each is not shaded? ________





3. On a square piece of paper, label the vertices in order A, B, C, and D. For the following problems, do not unfold the square between the steps. (When you are asked what percent remains, you are to give the percent still visible on one side.)

a) Fold A to B. What percent of the original square remains? _____
Fold B to C. What percent of the original square remains? _____

Unfold the paper and begin again.

b) Fold A to C. What percent of the original square remains? _____
Fold B to D. What percent of the original square remains? _____

c) Visualize to decide what percent of the square would remain after each of the following. Each part begins with the square unfolded. Fold the square to check your answers.

i) Fold A to the midpoint of side AB. What percent of the original square remains? _____

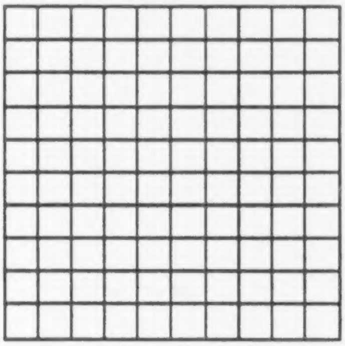
ii) Fold A, B, C, and D to the center of the square. What percent of the original square remains? _____

4. Shade $\frac{1}{2}$ of 1 percent ($\frac{1}{2}$%) of the first grid.

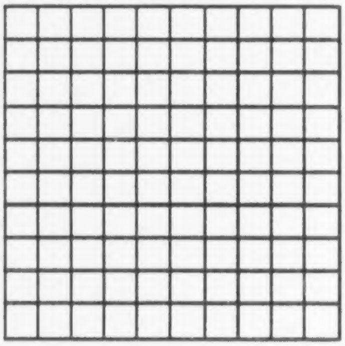
a) How many parts this shaded size are there in the whole grid? ____

b) Shade $\frac{1}{2}$ of the second grid.

Grid 1



Grid 2



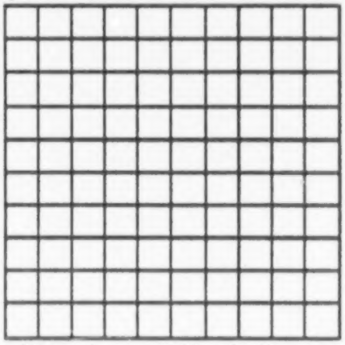
c) Explain the difference between $\frac{1}{2}$% and $\frac{1}{2}$.

d) What percent is shaded in Grid 2? _____

e) Since 1% is $\frac{1}{100}$, what fraction is $\frac{1}{2}$%? ___(Remember your answer in Part a.)

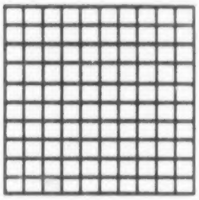
f) Express $\frac{1}{2}$% as a decimal. _________ (Remember what 1% is as a decimal.)

5. Shade 23.5% of the grid below in blue or in one shading. How many squares did you shade? ________
Shade 12.75% of this grid in pink or in a different shading. How many squares did you shade for 12.75%? ______

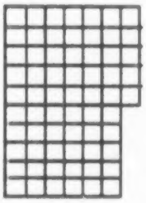


Determine (without counting) the number of unshaded squares. ____

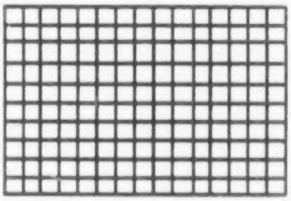
6. If Grid 1 below represents 1 whole, what percent is it? ___
Then what percent does the Grid 2 represent? ____
What about Grid 3? Is it more than 100% of Grid 1? ___
What percent is it? _______



Grid 1 100%



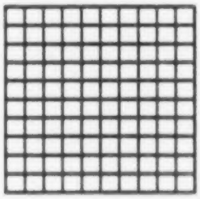
Grid 2 ____%

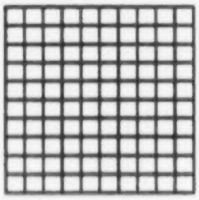


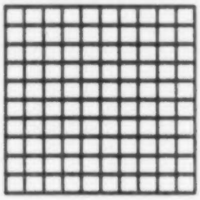
Grid 3 ____%

7. Complete the following, using diagrams as needed.
60 = ___ % of 100 80 = ___ % of 100 100 = ___ % of 100
120 = ___ % of 100 150 = ___ % of 100 200 = ___ % of 100

8. On each grid shade a design of your choice; then have a classmate first estimate the percent shaded, then determine exactly the percent shaded and the percent not shaded. You may use different colors and request the amount shaded in each color.







Estimated:	____% shaded	____% shaded	____% shaded
Exact:	____% shaded	____% shaded	____% shaded
Exact:	____% not shaded	____% not shaded	____% not shaded

USE YOUR PERCENT SENSE

1. List the coins you could use to make the following percents of a dollar using the fewest coins possible. Also write the amount in dollars and cents: $_.__

4%:

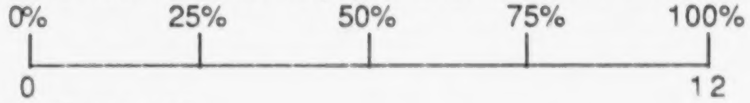
55%:

142%:

98%:

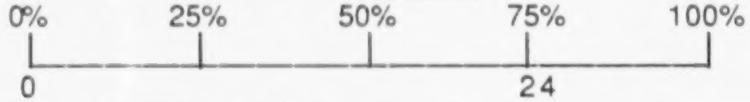
Give 4 ways to make 120% of a dollar.

2. Use the number lines to help you answer the following questions:



a) What is 75% of 12? ____

b) About how much is 30% of 12? ______

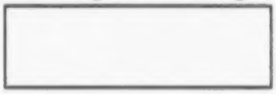


c) Twenty-four is 75% of what number? __________

d) What percent of that number is 8?___ About what percent is 10? ___

3. Finding 100% From a Part

a) This is 25% of a larger rectangle:

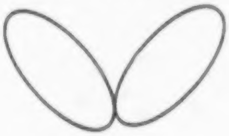


Draw 50% of the larger rectangle.

Draw 75% of the larger rectangle.

Draw 100% of the larger rectangle.

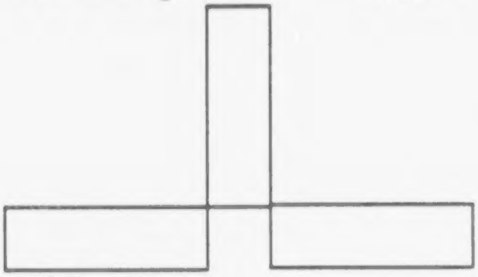
b) This is about 66% of a design:



Draw about 33% of the design.

Draw 100% of the design.

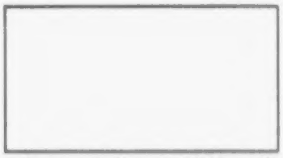
c) This is 75% of a figure. Shade 25% of the larger figure.



Draw 100% of the figure.

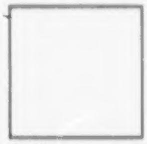
4. Finding 100% From 100% or More

a) This is 150% of a smaller rectangle. Shade 50% of the smaller rectangle.



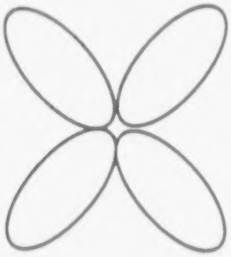
The unshaded part is 100% of the smaller rectangle.

b) This is 100% of a square. Draw 25% of the square.

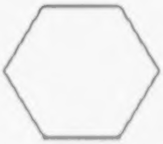


Draw 50% of the square. Draw 75% of the square.

c) This is 200% of a smaller figure. Shade the smaller figure.

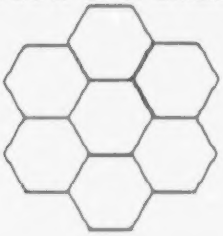


d) This is 120% of a smaller figure. Can you show 20% of the smaller figure?

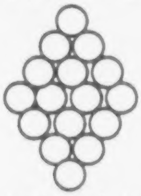


Shade 100% of the smaller figure.

e) This is 700% of a smaller design. Shade 100% of the smaller design.



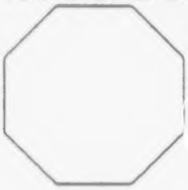
f) This is 160% of a smaller figure.



Draw 10% of the smaller figure.

Shade 100% of the smaller figure.

g) This is 160% of a smaller figure. Shade the smaller figure.



5. What Would It Be?

a) Would 90% of 5 be greater than 5, equal to 5, or less than 5? ____

b) Would 110% of 80 be greater than 80, equal to 80, or less than 80? __

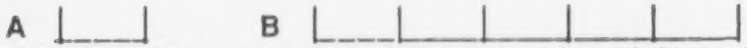
c) Would 10% of 130 be greater than 13, equal to 13, or less than 13? __

d) Would 1% of 234 be greater than 5, equal to 5, or less than 5? ____

e) Would 140% of 100 be greater than 140, equal to 140, or less than 140? ______

f) Would 0.5% of 100 be greater than 5, equal to 5, or less than 5? ____

6.

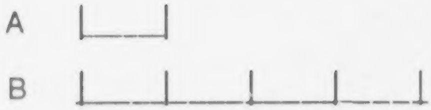


If **B is 100%** and B has 5 equal sections the size of A, each is 20%, so A is 20% of B.

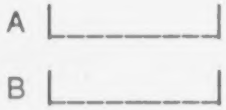
If **A is 100%**, since B is five times as long as A, B is 500% of A. Analyze each of the following pairs of segments in a similar way.

	A is ___% of B	B is ___% of A
a)		
b)	$33\frac{1}{3}$	
c)		
d)		

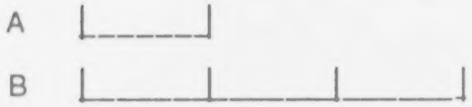
a)



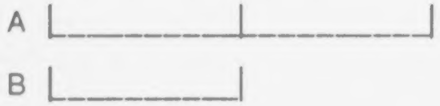
c)



b)



d)



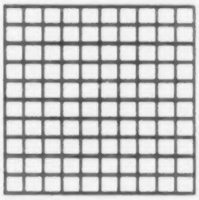
BENCHMARKS FOR ESTIMATING

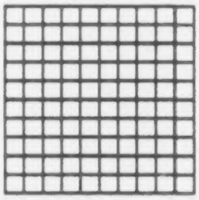
Some percents are more famous than others. It is useful to know their fractional equivalents for use in estimating and in mental computation.

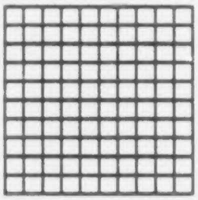
1. When you need to, shade a grid to help you find the **fractional equivalent** in simplest form for each of these percents.

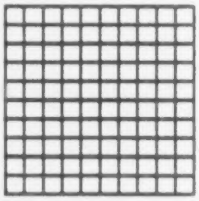
50% = _____ 25% = _____ $33\frac{1}{3}\%$ = _____ 75% = _____

$66\frac{2}{3}\%$ = _____ 12.5% = _____ 20% = _____ 10% = _____









2. Since percent means "per hundred," you can write the fraction with denominator 100 and simplify to find the fractional equivalent in simplest form. Show how to do this for the fractions above.

50% = __________________ 25% = __________________

$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$ 75% = __________________

$66\frac{2}{3}\%$ = __________________ 12.5% = __________________

20% = __________________ 10% = __________________

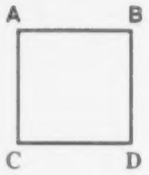
3. Justify your answers to the following with a diagram.

a) Is 0.6 closest to 0, $\frac{1}{2}$, or 1? ____ Write 0.6 as a percent. _____

b) Is 0.08 closest to 0, $\frac{1}{2}$, or 1? ____ Write 0.08 as a percent. _____

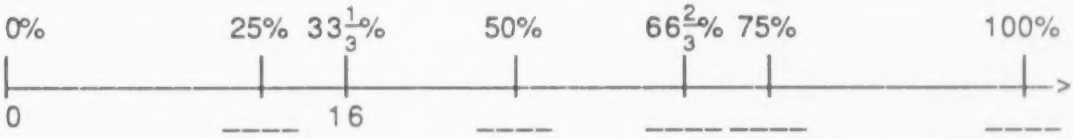
c) Is $\frac{3}{5}\%$ closest to 0, $\frac{1}{2}$, or 1? ____ Write $\frac{3}{5}\%$ as a decimal. _____

4. Visualize to decide what percent of the square you folded earlier would remain after folding A to C and then B to C. _____ Fold the square to check your answer.



5. Use the number lines to help you answer the following:

16 is $33\frac{1}{3}\%$ of _____ 50% of this number is _____



Write the numbers for the indicated percents of the number represented on this number line. What fraction of the number is 32? ______ **About** what percent of the whole is 5? ____

6. Visualize number lines and try to determine the following mentally:
   a) What is 25% of 40? _____
   b) What percent of 66 is 22? _____
   c) Thirteen is 50% of what number? ______
   d) What is $66\frac{2}{3}\%$ of 90? _____
   e) About how much is 78% of 40? ____
   f) About what percent of 88 is 45? _____
   g) Ten percent of what number is 10? _____
   h) Thirty is about what percent of 96? _____

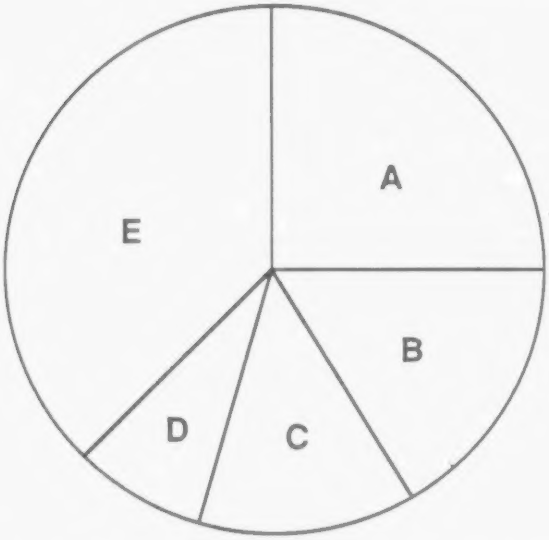
Decide whether you can easily get an exact answer mentally. If not, **estimate**.
   i) Two out of 20 is _____%
   j) Three is what percent of 11?
   k) Three is 75% of ____.
   l) Thirty-one percent of 92 is _______.

7. Estimate what percent of a meter each of the following is. Then measure for each part with string and compare the length to a meter stick for a more accurate estimate. Compare your results with your classmates. Discuss any unexpected differences.

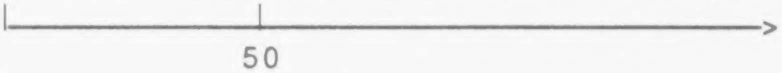
	Guess	Estimate using measurement
a) Your hand span	_____% of a meter	_____%
b) Your palm width	_____% of a meter	_____%
c) The length of your little finger	_____% of a meter	_____%
d) The length of your foot	_____% of a meter	_____%
e) Your arm span	_____% of a meter	_____%
f) Your height	_____% of a meter	_____%
g) The distance around your waist	_____% of a meter	_____%
h) The distance around your head	_____% of a meter	_____%

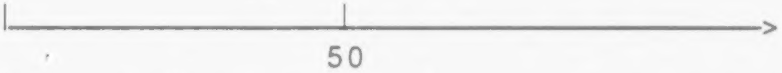
8. **About** what **percent** of the circle below is each piece?
A _____ B ______ C _____ D ______ E ______
What percent is the sum of your estimates? ___ Is this reasonable? ___

About what **fraction** of the circle is each piece?
A _____ B ______ C _____ D ______ E ______
What should the sum of these fractional parts be? _____



9. Show about where 100 would be on each line, using the number given.

a)


b)


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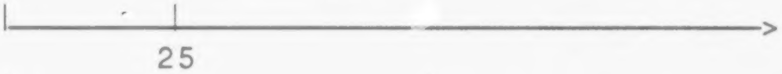
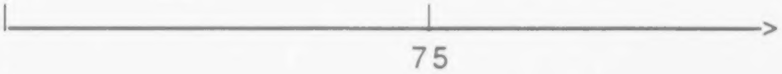
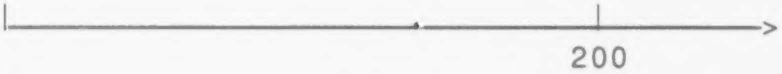


d)


e)


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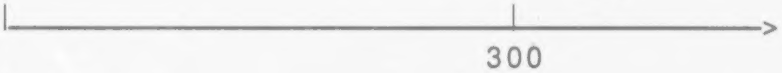
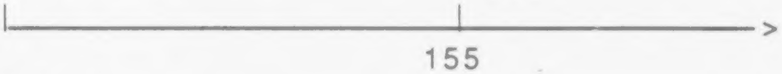


g)


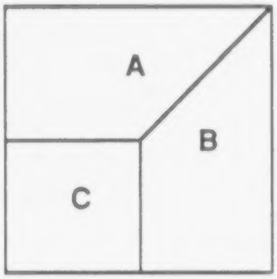
Make one up:

h)

10. For each diagram, use the letters to list the pieces in order from **smallest to largest**. Let the **smallest** piece be the whole (**100%**). Give the approximate percent of this whole that each of the other pieces will be.

a) Example

Smallest piece is C.

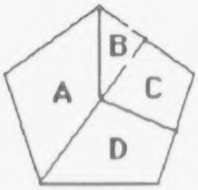


C is 100% of C

A is 150% of C

B is 150% of C

b)



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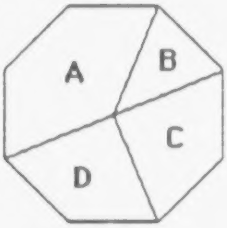

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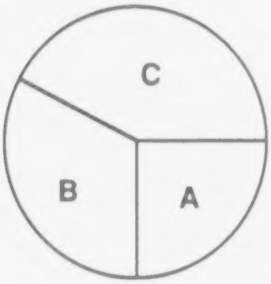

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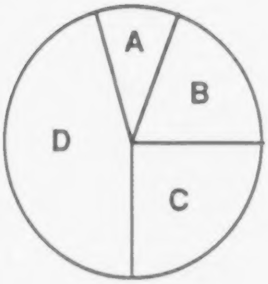


MENTAL COMPUTATION WITH PERCENTS

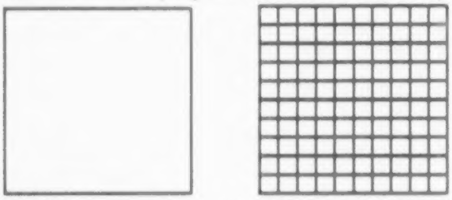
1. What is 10% of 50? ___
Explain how you can easily find 10% of any number.

How can you use this information to easily estimate a 15% tip?

If the tax on food eaten at a restaurant is 6.75% in your city, how can you use the amount of tax (written on your dinner check) to easily estimate a 15% tip?

Use your favorite method for estimating a 15% tip if the dinner check is $34.65 and the tax is $2.34 (6.75%). ____ Explain the method you used.

2. What fraction is equivalent to 80%? ___
If your club made 30 pizzas and sold 80% of them, find out how many pizzas they have left over. _______ Represent the pizzas using the empty square and the percent using the grid. Explain how these two diagrams can help you answer the question.



How would you use mental computation to answer the question?

3. Explain how you would estimate the cost of the following items with the discounts indicated. Use only mental computation.

a) A dress marked $49.99 at a 25% discount.

b) A CD marked $16.99 at 15% off.

c) A blouse at 75% off of the $18.99 clearance price.

d) A suit at 60% off the already marked-down price of $109.99.

4. If you have $30.00 left in your clothing budget this month, can you buy an outfit at Grand Finale marked $49.99 with a green tag ($33\frac{1}{3}$% off), or do you have to hope it is not sold before next month when it would be 50% off? Use mental computation and estimation only.

5. A bike originally priced at $149 is on sale for $99.95. About what is the percent discount? _____ How did you estimate?

6. If you paid $41.99 for a garment marked down 40%, about how much was it originally?

7. Draw two circle graphs to represent the following information and explain what it means.
The United States has only 5.7% of the world's population, but consumes 40% of the resources used in the world in a year. If the rest of the world had our standard of living, the known resources of the world would be exhausted in decades.

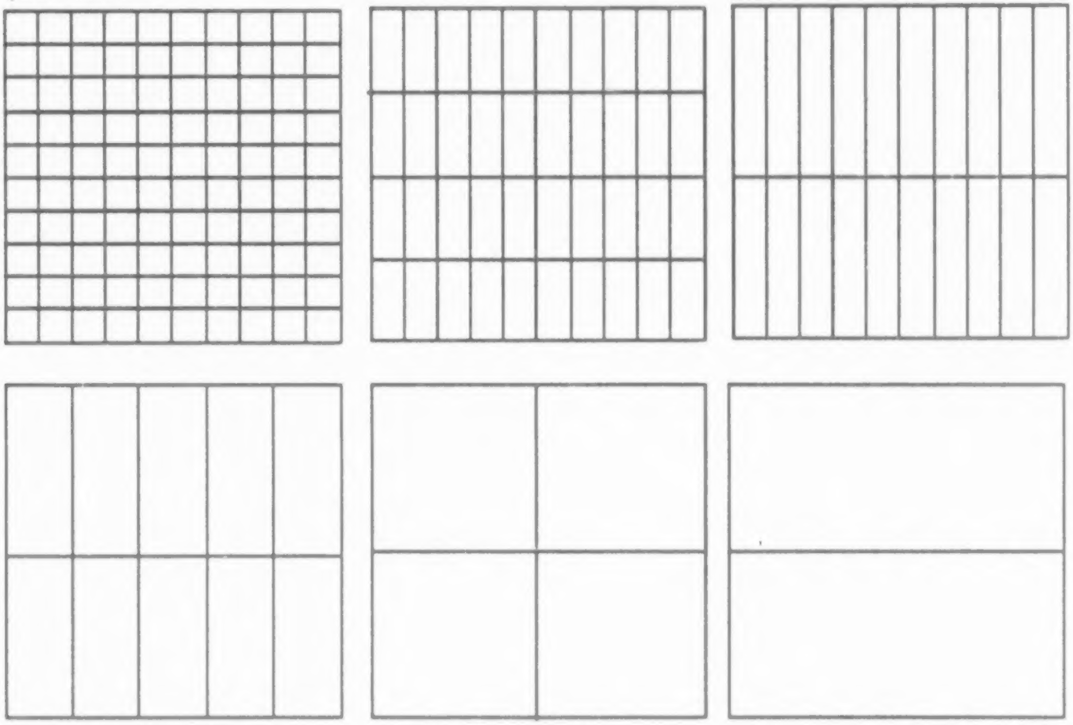
8. Explain whether this reasoning is valid.
"Our money in the bank was only making 6%. By withdrawing some to buy these golf clubs at 20% off, I made a 14% profit."

9. Andre answered 87% of the questions on his final exam correctly. If he answered 87 questions correctly, how many questions were there?

Explain your reasoning.

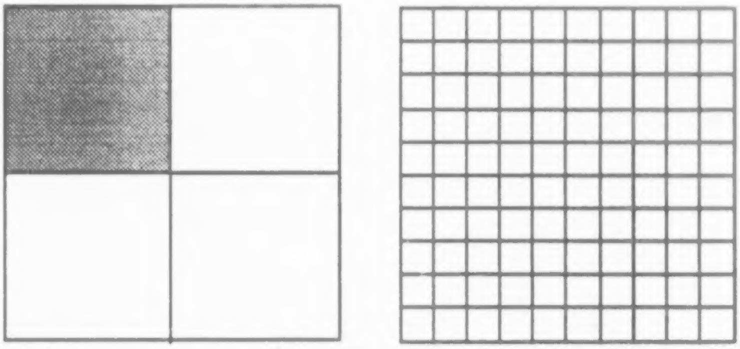
PROPORTION SENSE

1. Shade 50% of each of the following and use the results to answer the questions.



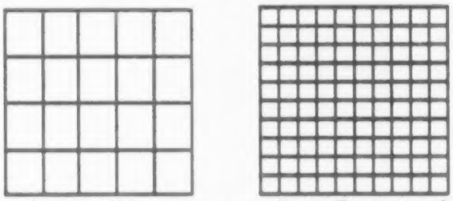
a) 50% of 20 is ____.
b) 50% of 40 is _____.
c) ____ is 50% of 100.
d) 50% of ___ is 1.
e) ____% of 10 is 5.
f) 2 is ___ % of 4.

2. Shade $\frac{1}{4}$ of the second grid to complete this equality: $\frac{1}{4} = \frac{\quad}{100}$



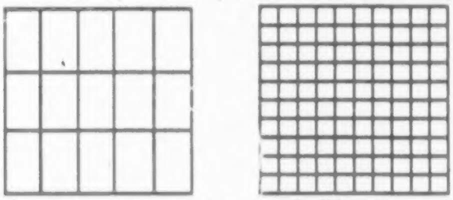
One shaded out of 4 is the same as ____ shaded out of 100.

3. Let's find what percent 5 is of 20. Explain how you can use the models (assume the wholes are the same size) to show that 5 is 25% of 20.



Explain how this means that 5 out of 20 is the same as 25 out of 100.

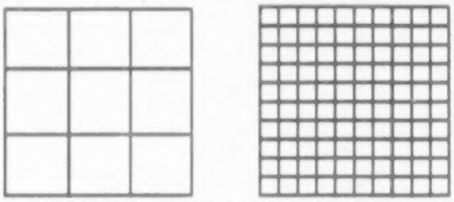
4. What percent of 15 is 3? Use these models (assume the wholes are the same size) to explain what the question means.



Three is twenty percent of fifteen. Use the models to explain why this is so.

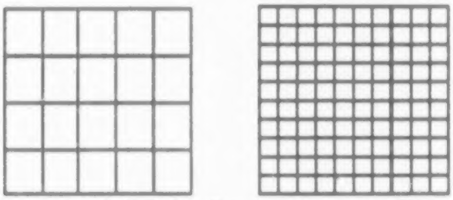
Therefore, three out of fifteen is the same as ___ out of 100?

5. About what percent of 9 is 4? Explain using the models.



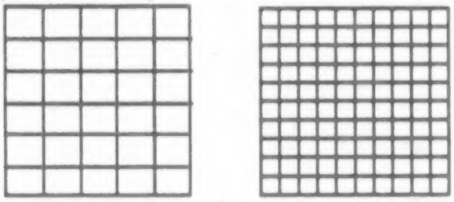
So, 4 out of 9 is about the same as ____ out of 100.

6. What is 75% of 20? Explain how you can use the models to show the answer.



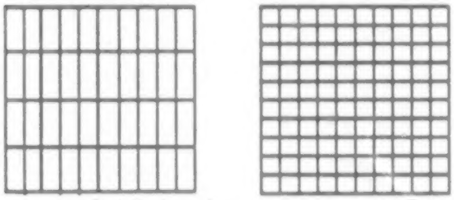
So ___ out of 20 is the same as 75 out of 100.

7. Explain how you can use the models to find 40% of 30.



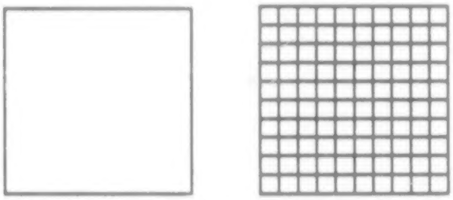
So 40 out of 100 is the same as ____ out of 30.

8. Use the models to find 15% of 40.



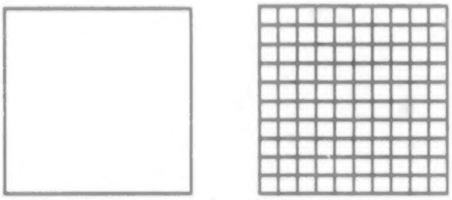
So ___out of 40 is the same as 15 out of 100.

9. Draw the model you would you use with the 100 grid to show:
50% of what number is 7?



So 7 out of ___ is the same as 50 out of 100.

10. Draw the model you would you use with the 100 grid to find 15% of what number is 30.

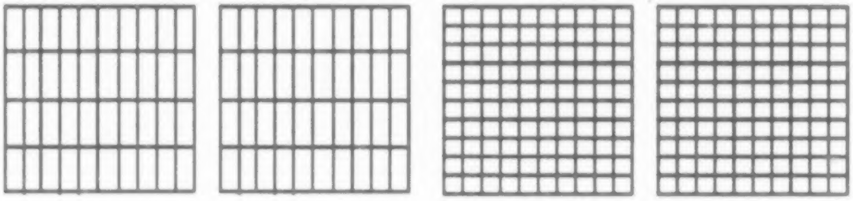


So ___ out of ____ is the same as _____ out of 100.

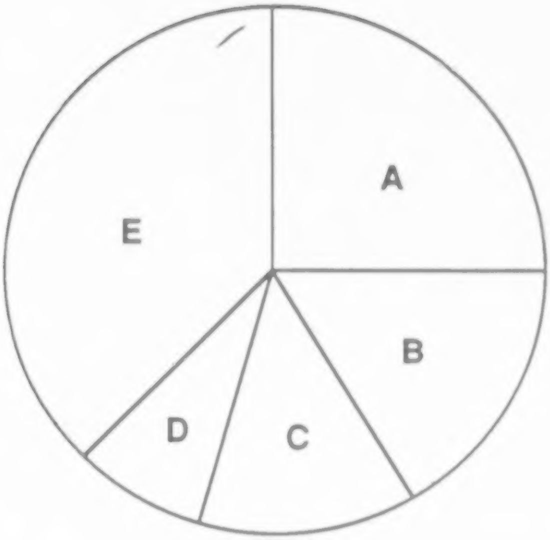
11. Find $\frac{1}{2}$% of 600.

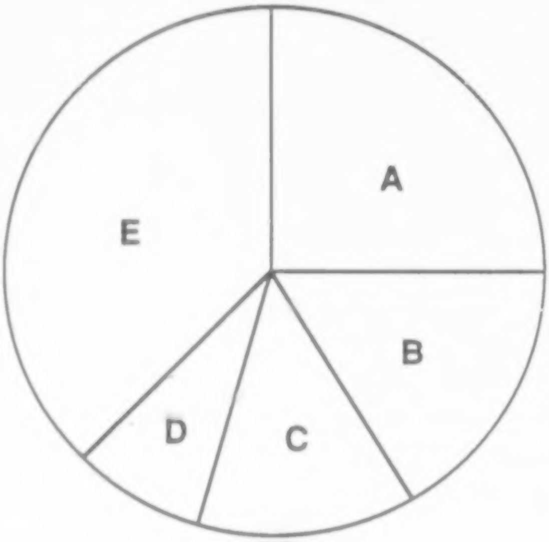
Explain how you found the answer.

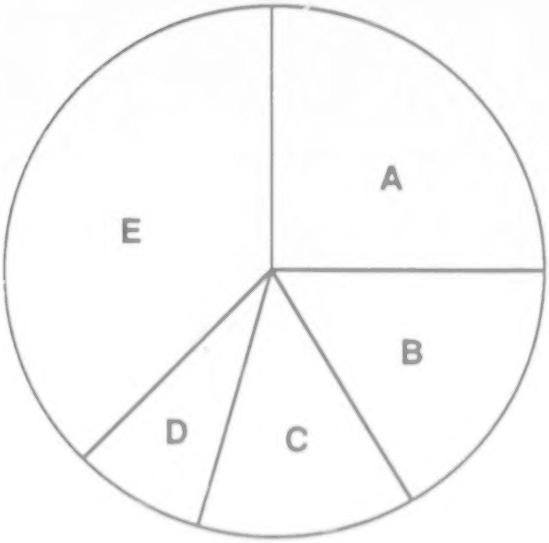
12. Is 150% of 40 more or less than 40? _____
Explain how to use these models to show how much 150% of 40 is.

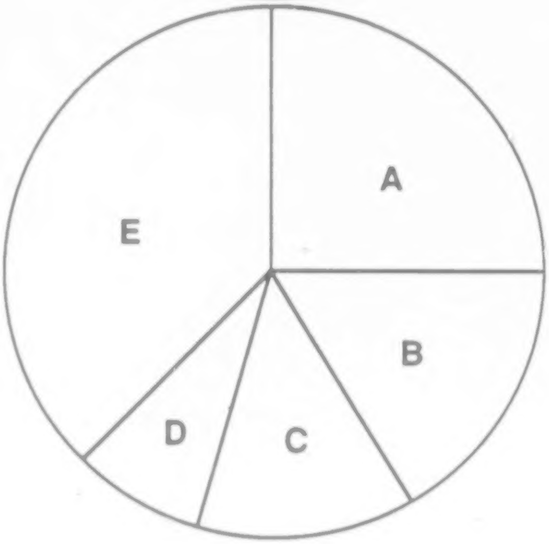


Transparency Master Part 3, Problem 8









RATIO SENSE

FOCUS: Meaning-Centered Lesson

- Ratio
- Rate

PURPOSE: The student will...

- Have an improved ratio sense, and
- Be able to represent a ratio or rate situation in a variety of ways.

TEACHER BACKGROUND: Instruction on ratio and proportion often goes too fast for many students. Researchers have noticed that even students who absorb a cross-multiply method of solving proportions may not trust the results and, if left to their own reasoning, may ignore that method. The four parts here are based on a research study which gave positive results.

MATERIALS: Showing Ratios and Rates 1, Ratios and Rates 2, 3, and 4 worksheets.

LESSON DEVELOPMENT: The intent is for the students to notice relationships, so there may be an advantage to using small groups with these lessons to maximize the opportunities to hear different approaches.

1. Showing Ratios and Rates 1. The focus here is on the easiest situation, that of a unit ratio or unit rate (second quantity = 1). The exercises set the stage for a multiplicative comparison of like quantities. You may wish to continue the entries in the tables.

2. Ratios and Rates 2. The given ratios are designed to encourage the students to find the unit ratio first. The class discussion should put this informally in words, perhaps something like "If you can find out how much per one, then the problem is easy."

3. Ratios and Rates 3. These ratios are chosen to encourage a division or a fractional approach. The discussion could include mention, with examples, of the three ways of thinking so far.

4. Ratios and Rates 4. The last ratio in each problem may be harder to think through; the unit ratio is not an "easy" number. Hence, it is especially important to plan for a discussion of the students' ways of thinking. Worth discussing also is that in ratio settings, exact answers (like 9 1/3 girls) may give numbers that have to be interpreted in the setting of the problem. A ratio like 1.6 children per family might correctly describe the ratio in simplest form, even though a family actually having 1.6 children is ridiculous.

EXTENSIONS: You will note that all the exercises on these pages have the unknown quantity in the first position. The idea can easily be adapted to having the unknown be the second quantity (this was done in the research study).

ANSWERS:
Showing Ratios and Rates 1.
1. 6 cakes, 9 cakes   2. $8.45, $13.52   3. 188.4 miles, 282.6 miles
4. 286.5 Calories, 687.6 Calories

Ratios and Rates 2.
1. 12.6 miles   2. 100¢, 125¢   3. 6 minutes, 14 minutes

Ratios and Rates 3.
1. 5.3 pounds   2. $9.20   3. 6 cups   4. 23¢

Ratios and Rates 4.
1. 8 girls, 9 1/3 girls   2. 180 push-ups, 157 1/2 push-ups
3. 6 hits, 10 1/2 hits

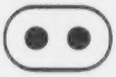
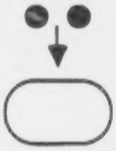
SOURCE: Development suggested in
Case, Robbie. (1985). *Intellectual Development.* Orlando, Fl: Academic Press.

SHOWING RATIOS AND RATES 1

Apples cost $1.89 for 2 pounds.
The factory puts 2 cupcakes in 1 package.

You can describe such situations with ratios. A **ratio** is a comparison of two quantities, like the $1.89 and the 2 pounds. If the quantities are two different kinds, like money and pounds, a ratio is often called a **rate**.

You can show a ratio or rate in several ways. For example, here are different ways of showing that the factory puts 2 cupcakes in 1 package:

A. Diagram:  or 

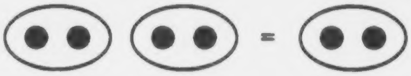
B. Words: 2 cupcakes in 1 package, or 2 cupcakes in a package, or
2 cupcakes for every package, or 2 cupcakes for each package, or
2 cupcakes per package, or the ratio of cupcakes to packages is 2 to 1.

C. Symbols: 2:1, or $\frac{2}{1}$ (Much of the time you can do the same things with ratios as you can with fractions.)

D. Table:

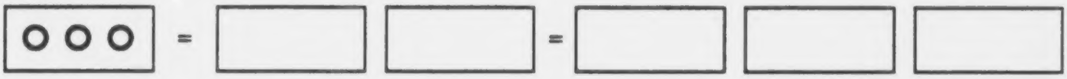
# cupcakes	2				
# packages	1				

Since 4 cupcakes in 2 packages involves the same relationship as 2 cupcakes in 1 package--in each case there are 2 cupcakes per package--we say the two ratios are **equal** and write, as with fractions, $\frac{4}{2} = \frac{2}{1}$, or sometimes 4:2 = 2:1. When we show this with diagrams, as below, the equals sign means, "is the same ratio as."



Complete.

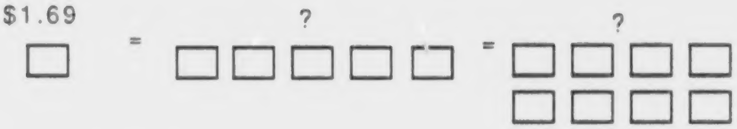
1. 3 cakes per package = _?_ cakes per 2 packages = _?_ cakes per 3 packages



$$\frac{\text{(cakes)}}{\text{(packages)}} \quad \frac{3}{1} = \frac{?}{2} = \frac{?}{3}$$

# cakes	3	?	?		
# packages	1	2	3		

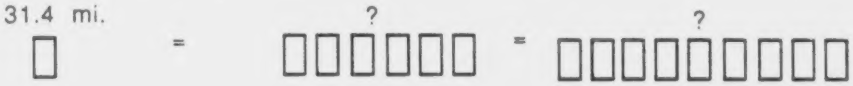
2. $1.69 per lb. = _____ per 5 lbs. = _____ per 8 lbs.



$$\frac{\$1.69}{1} = \frac{?}{5} = \frac{?}{8}$$

amount of money	$1.69	?	?		
# of lbs.	1	5	8		

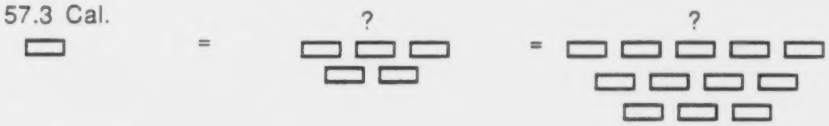
3. 31.4 mi. per gal. = _____ mi. per 6 gal. = _____ mi. per 9 gal.



$$\frac{31.4}{1} = \frac{?}{6} = \frac{?}{9}$$

# miles	31.4	?	?		
# gallons	1	6	9		

4. 57.3 Calories per g = _____ Cal. per 5 g = _____ Cal. per 12 g



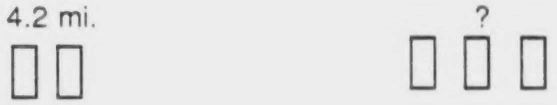
$$\frac{57.3}{1} = \frac{?}{5} = \frac{?}{12}$$

# Cal.	57.3	?	?		
# grams	1	5	12		

RATIOS AND RATES 2

Complete.

1. Jog 4.2 mi. every 2 days = jog ____ mi. every 3 days

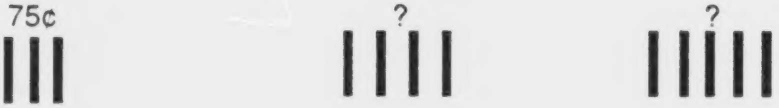


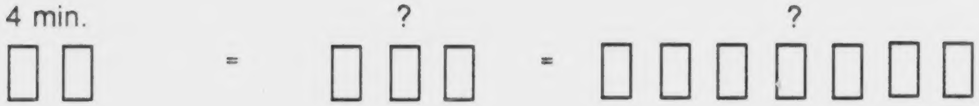
$$\frac{4.2}{2} = \frac{?}{3}$$

# miles	4.2	?			
# days	2	3			

2. 75¢ per 3 pencils = ______ per 4 pencils = ______ per 5 pencils

$$\frac{75¢}{3} = \frac{?}{4} = \frac{?}{5}$$



3. 

4 min. for 2 pages = ____ min. for 3 pages = _____ min. for 7 pages

$$\frac{4}{2} = \frac{?}{3} = \frac{?}{7}$$

# minutes	4	?	?		
# pages	2	3	7		

RATIOS AND RATES 3

Complete.

1. 10.6 lb. = ?

☐ ☐ ☐ ☐ = ☐ ☐

10.6 lb. for 4 boxes = ___ for 2 boxes

$$\frac{10.6}{4} = \frac{?}{2}$$

2. $18.40 for every 6 hours = _____ for every 3 hours

$18.40 = ?

• • • • • • = • • •

$$\frac{18.40}{6} = \frac{?}{3}$$

3. 12 cups for 8 dogs = _____ cups for 4 dogs

12 = ?

• • • • • • • • = • • • •

$$\frac{12}{8} = \frac{?}{4}$$

4. 69¢ per 6 eggs = _____ per 2 eggs

69¢ = ?

○ ○ ○
○ ○ ○ = ○ ○

$$\frac{69}{6} = \frac{?}{2}$$

# cents	69	?		
# eggs	6	2		

RATIOS AND RATES 4

Complete.

1. 4 girls per 3 boys = ____ girls per 6 boys = _____ girls per 7 boys

□□□□ = ? = ?

△△△   △△△△△△   △△△△△△△

$$\frac{4}{3} = \frac{?}{6} = \frac{?}{7}$$

2. 90 push-ups in 4 min. = ____ push-ups in 8 min. = ____push-ups in 7 min.

90   ?   ?

M M M M   M M M M M M M M   M M M M M M M

$$\frac{90}{4} \qquad \frac{?}{8} \qquad \frac{?}{7}$$

# push-ups	90	?	?		
# minutes	4	8	7		

3. 3 hits per 10 at-bats = ____ hits per 20 at-bats = ____ hits per 35 at-bats

• • •   ?   ?

A A A A A A A A A A = A A A A A A A A A A = A A A A A A A A A A

A A A A A A A A A A   A A A A A A A A A A

A A A A A A A A A A

A A A A A

$$\frac{3}{10} = \frac{?}{20} = \frac{?}{35}$$

# hits	3	?	?			
# at-bats	10	20	35			

PUTTING IT TOGETHER

FOCUS: Meaning-Centered Lesson for Addition
- Put Groups/Amounts Together
- Missing Addend
- Writing Equations

PURPOSE: The students will...
- Be able to use drawings and word problems to link groups or amounts with addition.

STUDENT BACKGROUND: Students should have a number sense of whole numbers and decimals. They also need to know metric conversions.

MATERIALS: PUTTING IT TOGETHER worksheet.

LESSON DEVELOPMENT: The teacher might open the lesson with a discussion of addition. Addition can be expressed in two specific ways. Primarily, addition is linked to grouping things together as illustrated in Problems 1 and 2. The groups or amounts may be literally combined as in Problem 1, or mentally combined, as in Problem 2. (Research shows that problems of these two types are not equally easy for younger students). Problems 3 and 4 group things together but subtraction is used to find the solutions. Having the students write equations can be helpful to underscore the fact that these are both grouping problems. The first two are solved by using addition and the last two use subtraction. Drawings can also be a useful tool to help students see the subtle distinctions.

ANSWERS:

1a) see drawing
b) $210.00
c) $40 + $45 + $50 + $75 = $210.00

2a) see drawing
b) 107 centimeters or 10.7 decimeters
c) 27 + 24 + 30 + 8 + 6 + 5 + 7 = 107

3a) see drawing
b) 2 ft 8 in
c) 8 = 5' 4" + X

4a) see drawing
b) 7 RAP albums
c) 19 = 12 + R

PUTTING IT TOGETHER

1. You have just arrived at Magic Mountain with three other friends. Your mom and another parent have told you where to meet at 6:00pm, and you have decided to pool all your money. You have $40, Minh has $45, Rosa has $50, and Dennis has $75. How much money do you have in your pool?

a) Make a drawing in the space provided:

b) Solve the problem: ____________

c) Write an **EQUATION**: _______________

2. Bob would like to make a file cabinet. Each drawer is a different height. One is 2.7 decimeters, another is 2.4 decimeters, and the third is 3 decimeters. If there is a strip of wood on the bottom that is 8 centimeters high, one between the second and third drawer that is 6 centimeters high, another between the first and second drawer that is 5 centimeters high, and one across the top that is 7 centimeters high, how tall will Bob's file cabinet be? (Be sure to make all units the same)

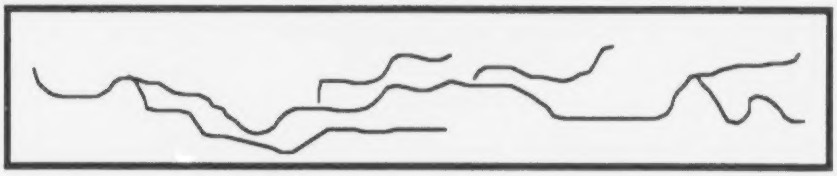
a) Make a drawing in the space provided:

b) Solve the problem: ____________

c) Make an **EQUATION**: ____________________

3. In the wall in the game room, there is an 8 foot horizontal crack that Lynn wants to cover. Since she does not have one-eight foot board, she must put two pieces together. The first piece is 5 feet 4 inches. How long must the other one be to cover the crack in the wall?

a) Make a drawing to cover the crack in the wall:



b) Solve the problem: ____________

c) Write an **EQUATION**: _______________

4. Nguyet has 19 CD's since she put her RAP albums with her soul albums. If she had 12 soul CD's, how many RAP CD's did she have to to add to her collection?

a) Make a drawing:

b) Solve the problem: _______________

c) Write an **EQUATION**: _________________

TAKE IT AWAY AND COMPARE

FOCUS: Meaning-Centered Lesson

- Take Away
- Comparison
- Missing Addend

PURPOSE: Students will...

- Understand the meanings of subtraction in problem solving; and
- Will be able to write appropriate equations.

TEACHER BACKGROUND: Research indicates that students can do take-away problems effectively, but have difficulty with subtraction situations that involve missing addends and, especially comparisons. This lesson provides opportunity for students to work with all of these situations and make distinctions among them.

MATERIALS: TAKE IT AWAY AND COMPARE worksheet, and a calculator.

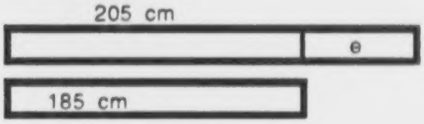
LESSON DEVELOPMENT: Teacher needs to stress distinctions among take away, comparison, and missing addend. This can be made especially clear with illustrations. Drawings for take away will focus on removing a part from a whole. In contrast to this, a drawing for a comparison situation will involve two items. Specific attention will be paid to: How much bigger? How much less? or How much more is needed to be equal?

Missing addend problems may require more attention, especially since some students may see it as an addition problem.

There will need to be some discussion about solving equations. An idea that might involve some difficulty will be a subtracted variable if/when it appears.

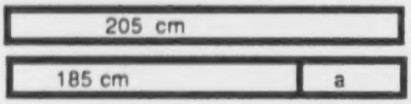
The first three problems are take away situations. Problem 4 is the first comparison situation. The teacher should be prepared to discuss two possible drawings and make distinctions between comparison and missing addend:

Comparison



185 cm = 205 cm - e, or
205 cm - 185 cm = e

Missing Addend



185 cm + a = 205 cm

Some students may see Problem 5 as a missing addend situation, or they might see it as a comparison situation. The teacher should be prepared for a discussion of either.

ANSWERS:

1a) Drawings will vary
 b) 1 - 3/4 = Y
 c) 3 yards will be used

2a) Drawings will vary.
 b) T - $6.24 = $11.05 or T - $11.05 = $6.24, or $11.05 + $6.24 = T
 c) $17.29 originally

3a) Drawings will vary
 b) $2 - \frac{3}{4} = B$
 c) $1\frac{1}{4}$ pizzas for breakfast

4a) Drawings will vary.
 b) 205 - 185 = L, or 205 - L = 185, or 185 + L = 205
 c) 20 cm longer

5a) Drawings will vary.
 b) $1246.79 + N = $1324.51, or $1324.51 - $1246.79 = N
 c) $77.72 is needed

6a) Drawings will vary.
  b) 1.3% - D = 0.1%, or 1.3% - 0.1% = D
  c) 1.2% fuel is lost in Discovery

7a) 185 cm + N = 205 cm
  b) 20 cm

8a) \$1324.51 - M = \$1246.79, or \$1246.79 + M = \$1324.51, or
\$1324.51 - 1246.79 = m
  b) \$77.72

TAKE IT AWAY AND COMPARE

1. Aileen wanted to make some biking shorts. She bought one yard of fabric. If the pattern calls for three-quarters of a yard how much was excess?

a) Draw a picture that illustrates the problem:

b) Write an equation that represents your illustration __________

c) Solve the problem: ________

2. If Larry gave Bonnie $6.24 for lunch, and he still had $11.05, how much did he have originally?

a) Draw a picture that illustrates the problem:

b) Write an equation that represents your illustration: __________

c) Solve the problem

3. Frances ordered 2 pizzas but was only able to eat three quarters of one of them. How much cold pizza would she have left for her breakfast?

a) Draw a picture that illustrates the problem:

b) Make an equation that represents your illustration: __________

c) Solve the problem: ____________

4. Larry bought skis that were 205 cm. Tom's skis were only 185 cm. How much longer were Larry's skis than Tom's? Try solving this problem mentally, then:

a) Draw a picture that illustrates the problem:

b) Write an equation that represents your illustration: __________

c) Solve the problem: ______________

5. Your school was having a drive to earn money for the needy. You were competing with a neighbor school by saving glass, aluminum and plastic. So far, they have earned $1324.51. You have earned $ 1246.79. How much do you need to earn to catch up with them?

a) Draw a picture that illustrates the problem:

b) Write an equation that represents the illustration: __________
c) Solve the problem: ___________

6) NASA has a fuel problem with its shuttle missions. The Atlantis was losing 1.3% of its fuel on the launching pad. This was 0.1% more than Discovery. What percent fuel loss did the Discovery experience?

a) Make a drawing to illustrate the problem?

b) Write an equation that represents your illustration: ___________
c) Solve the problem: ___________

7. Reread Problem 4. Write an equation and solve this question:
How many more centimeters would Tom's skis need to be the same length as Larry's? EQUATION: ________________
ANSWER: _____cm

8. Reread Problem 5. Write an equation and solve this question:
How much more did the neighbor school earn than your school?
EQUATION: ________________ ANSWER: $_________

ADDING AND SUBTRACTING FRACTIONS

FOCUS: Application Activity
- Putting amounts together
- Take-away situation

PURPOSE: The students will...
- Work with fractions in different settings; and
- Have an opportunity to generalize what they have learned.

STUDENT BACKGROUND: Students will need to be proficient adding and subtracting fractions before they begin this activity. Terms that may require definiton are threshold, molding, frame, and French doors.

TEACHER BACKGROUND: Even though students may have relative computational successes when selectively dealing with addition or subtraction of fractions, it becomes more difficult when they must reason through their reading and subsequently decide for themselves what operation must be performed. This lesson focuses on adding and subtracting fractions and does not always provide the standard key words to signal any operational strategies.

MATERIALS: ADDING AND SUBTRACTING FRACTIONS worksheet

LESSON DEVELOPMENT: Problem 1, if assigned, should be used as a refresher and a warm up. Questions and discussion about the nature of putting groups/amounts together and taking amounts away can be effective.

EXTENSION: As a **FOLLOW UP**, ask the students to work in groups to solve the last problem. Working with peers might be especially beneficial because the strategy and the solution may be vague. Working together, they should be more confident that what they have arrived at is viable, especially since they have to defend their answer to the class.

ANSWERS:

WARM UP ANSWERS:

1a. 16 blocks
  b. 7/16
  c. 11/16   (7/16) + (4/16) = 11/16
  d. 3/16   (11/16) - (8/16) = 3/16

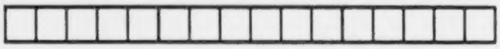
ANSWERS TO LESSON:

1a) Estimate: 6' 7" (answers may vary)
  b) Exactly: 6 ' 6 3/4 "
1c) Answers may vary
2a) 6 3/4"
  b) Answers may vary
  c) Answers may vary

EXTENSION ANSWERS:

Write 2 fractions: may vary
Explanation: may vary
Are there more fractions that work? YES
Generalization: $0 < fs < f_L$; and $1/4 < f_L < 1$; and
$f_L - fs < 1/4$; and $f_L + fs > 1/2$

STUDENT **WARM UP**

1. 

a) How many blocks are in this figure?_____

b) Shade seven of the blocks. Now what portion of the entire figure is shaded? _____

c) Shade four more blocks. Now how much of the figure is shaded? _____ Write an equation _________

d) If you erased eight shaded blocks what portion would be shaded? _____ Write an equation _________

ADDING AND SUBTRACTING FRACTIONS

1 a) A carpenter was building a door frame for French doors. It was six feet wide and 6 feet 9 inches high. The threshold was 8 inches wide. If he wanted to put a 3 $^{3/8}$ inch strip of molding around the frame, ESTIMATE how wide the door frame would be: _______. Sketch the problem in the space provided:

b) How wide is it exactly? _______

c) Explain how you arrived at your answer. Be sure to include what operation you used and why you think your answer is correct.__________

____

____

2 a) Look at the sketch you made so you can visualize what the doors would look like with the molding attached. You should be able to see that the frame is wider now. How much wider? _____

b) Explain how you found your answer.____________________________

____

____

c) Explain another way you might determine the answer.____________

____

____

EXTENSION EXERCISE

1. Can you find two fractions (bigger than zero and less than one) to add together that will be more than one-half? Also consider that if you subtract the smaller fraction from the larger one, the result is less than one-fourth. Write your two fractions: _____ &_____. Explain how you arrived at your answer.____________________

____

____

____

____.

Are there other fractions that work? _______

Can you describe in a more general way, all the fractions that will work? ____

____

____

IS THAT TRUE?

FOCUS: Meaning-Centered Lesson for Multiplication
- Equal groups
- Proportions

PURPOSE: The student will...
- Interpret results in situations where mathematical results must be carefully interpreted;
- Use proportions and/or equal-groups multiplication to solve word problems; and
- Provide explanations for their solutions.

STUDENT BACKGROUND: Students need to know how to solve proportions, and (if calculators are not available) how to multiply and divide decimals.

MATERIALS: IS THAT TRUE? worksheet

LESSON DEVELOPMENT: Students may choose to solve the word problems by using proportions or by using equal groups. Students should share their solutions and especially their explanations with examples of both methods with the class. Encourage students to consider the context when reaching an answer. In these situations, to use mathematics will give answers that are probably not realistic. All four questions are open-ended questions and are open to the student's interpretation. All resonable answers should be accepted; the explanations are most important. Problem #2 especially lends itself to many interpretations. Some students may feel more information is needed.

EXTENSION: The extension is a small group exercise. Word problems are then exchanged and solved by other groups.

ANSWERS: 1. a) 9 + 12 = 21 tries. b) No, answers will vary. c) Answers will vary. 2. a) Possible answer: 61.95 + 61.95 + 30.98 = 154.88. b) Answers will vary. 3. a) Possible answer: 128 sec. b) Answers will vary. c) Answers will vary.

IS THAT TRUE?

1. If it takes you three tries to make one basket from 20 feet away from the goal, and four tries to make one basket from 30 feet away from the goal, how many tries will it take you to make six baskets, three from 20 feet and three from 30 feet?

a) Answer:____________________________

b) Is this the only answer?

Explain.____

____

c) How many tries will it take you to make one basket from 50 feet away from the goal?

Explain.____

____

2. Your parents give you Nintendo for your 14th birthday. The next day you go shopping for video games at the discount store. The games are on sale; You can purchase any two games for $61.95. Since you received lots of cash, you decide to buy five games. How much do you spend?

a) Answer:__________________________

b) Is this the only answer?

Explain.____

____

3. In physical education class next week you will be running the 800-meter run. Last week, you ran the 200 meter run in 32 seconds. Your best friend ran the 200 m in 35 seconds. How long will it take you to run the 800-meter run?

a) Answer:

b) Is this the only answer?

Explain.____

____

____

____

c) Will your best friend do better than you? Explain.

____

____

____

____

4. Does calculating an answer mathematically--only working with the numbers--always give the best or only answer to the word problem? What else must you consider when finding an answer? Explain.____

____

____

____

EXTENSION: Write a similar word problem that illustrates what you have learned in this lesson.

PIZZA PARTY

FOCUS: Application Activity for Multiplication

- Equal groups

PURPOSE: The student will . . .

- Use equal-groups multiplication with fractional multiplicands, or addition of fractions with and without diagrams;
- Use diagrams to solve or justify solutions in these situations;
- Interpret fractional and decimal remainders meaningfully;
- Make decisions and incorporate the results in problem solving; and
- Manufacture data for use in some problems.

STUDENT BACKGROUND: Students should have fraction sense.

MATERIALS: PIZZA PARTY worksheet

LESSON DEVELOPMENT: Students may decide whether one person should have some pizza with a second-choice topping to eliminate the purchase of one pizza.

ANSWERS:

1.

Topping Cost per pizza	Vegetarian $7.40	Pepperoni $8.15	Sausage/onion $8.60	Canadian bacon $8.15
# of orders	1 person	5 people	3 people	4 people
Diagram				
Amount of pizza	3 / 4	3 3/4	2 1/4	3
Pizzas to order	1	4	3	3
Cost	$7.40	$32.60	$25.80	$24.45

Total cost of pizza is $90.25
If one person who ordered sausage and onion ate 1/4 of a vegetarian or a pepperoni pizza, $8.60 could be saved.

2. 8 x 2/3 is 5 1/3 so 6 bottles of cola are needed.
   5 x 2/3 is 3 1/3 so 4 bottles of noncola are needed.
   Cost of 10 bottles is $9.90.
   Ninety-nine cents could be saved if one person drinks some soda of each kind.

3. Answer depends on amount of pie to be allowed per person.
4. Cost of pizza + Cost of soda + Cost of pie = Cost so far
5. Answers vary. Students could decide on 1 1/2 of some fruit each.
6. Cost of pizza + Cost of soda + Cost of pie + Other Cost = Total Cost
   Total Cost ÷ 13 = Cost Each
7. $\frac{\text{Total Cost - \$60}}{13} = \frac{\text{Cost for group members}}{13} = \text{Cost each}$
8. Cost for all thirteen group members ÷ 12

or

$$\text{Cost each} + \frac{\text{Cost each}}{12} = \text{New Cost Each}$$

PIZZA PARTY

To celebrate being named Citizens of the Month for this quarter, those chosen will have a pizza party. Thirteen people will attend (unless you are superstitious and want to invite one more person). Your job is to calculate the cost of the food and decide how much each person will contribute to purchase it.

1. The following table gives information on the advance pizza orders and the cost per pizza at Parcheesi's. **Each person is to be allowed $\frac{3}{4}$ of a medium pizza and will choose one of the toppings offered.** Use diagrams to find the amount of pizza with each topping you will need to order or to justify the answer you get another way. Decide on the number of pizzas with each topping to order and the cost for each kind. Then find the cost of all the pizza for the party.

Topping Cost per pizza	Vegetarian $7.40	Pepperoni $8.15	Sausage/onion $8.60	Canadian bacon $8.15
# of orders	1 person	5 people	3 people	4 people
Diagram				
Amount of pizza				
Pizzas to order				
Cost				

Cost of pizza for the party: ___________

How much money could be saved if one person is willing to have some pizza with a different topping from the person's first choice? _____
Explain ____
____
____

2. You have decided to allow $\frac{2}{3}$ of a 2-liter bottle of soda for each person; 8 want cola, and 5 want noncola. How many bottles of each should be purchased ? You may use diagrams to get your answer or to justify it. If the soda is on special for $0.99 per bottle, find the cost.

Cost of soda for the party: __________

How much money could be saved if one person is willing to have some of each kind of soda? __________
Explain ____
____
____

3. For dessert, the majority voted for strawberry pie that costs $7.90 per pie at the Perfect Pie Shop. Decide what part of a pie to allow for each person. __________
How many pies will you order for the thirteen people? __________
How did you determine the number of pies you need? ______________
____

4. What is the cost of the refreshments (including the pie) decided upon so far? _____ Write the equation.

5. Decide what other refreshments to have. Include the unit price, quantity, and estimate the total cost. Include something nutritious that you will allow $1\frac{1}{2}$ of per person. Find the total amount needed for the 13 people and the cost.

Kind of Food
Amount per person
Total amount
Unit cost
Total estimated cost

6. Determine the total amount each person will pay and describe how you found it.

7. The ASB has heard about the party you are planning. They voted to contribute $60.00 to help pay for the refreshments. Now how much is each person's share?

8. One member of the group became ill just before the party, after all of the food had been purchased. Who should pay for that share?

If you decide to have the remaining people share the cost, explain two ways you could determine the new cost for each person.

FRACTIONAL PARTS

FOCUS: Meaning Centered Lesson for Multiplication
- Part of a group

PURPOSE: The student will...
- Use pictures to understand the mathematical relationship; between the numbers and the answer;
- Make generalizations about the applications of multiplication and especially the meaning of multiplication in fractional settings; and
- Use the appropriate language when giving a reason for the use of a particular operation.

STUDENT BACKGROUND: Student needs to know the basic computation of all operations with fractions and be familiar with the language: change, missing addend and equal amounts.

LESSON DEVELOPMENT: Word Problem: Each page of Janet's coin collection book holds 24 dimes. On one full page, $\frac{1}{3}$ of the dimes are LIberty Heads.

Questions. Ask students to generate as many questions about this problem as possible.

Suggestions:
* 1. How much are the 24 dimes worth?
  2. How many pages are in the collection book?
* 3. What does a Liberty Head look like?
  4. Is $\frac{1}{3}$ greater than one or less than one whole?
* 5. What fraction of the page is not Liberty Heads?
  6. What is $\frac{1}{3}$ of 24?
* 7. How many dimes are Liberty Heads?

*Teacher elicits the *questions if not given.

- Go through list of questions and discuss which can be answered and which need more information than that provided in the word problem.
- Select three questions to answer.
- Ask students to make pictures to illustrate the solution to each question.

PARTS OR GROUPS

FOCUS: Application Activity for Multiplication
- Equal groups or amounts
- Part-of-a-group or amount

PURPOSE: The student will...
- Know that multiplying the number in equal groups/amounts by a whole number gives the total amount, and
- Know that multiplying by a fraction or decimal (less than one) gives part of an amount or group.

STUDENT BACKGROUND: Students need to know what a leg in a foot race is. Students will need to know percents and discounts.

TEACHER BACKGROUND: Students often have the misconception that multiplication always makes bigger. This lesson will provide ample opportunity for students to see that multiplying by a fraction or decimal (<1) does not make bigger.

MATERIALS: PARTS OR GROUPS worksheet

LESSON DEVELOPMENT: Since this lesson will include recognizing extraneous information, some attention should focus on how to identify unnecessary information and discard it.

Problem 4 may take some discussion regarding the meaning of a 5.5% discount. It may also require some discussion about rounding money.

Each problem addresses the multiplying misconception by asking the students to determine whether the solution will be larger than the whole. Most of the teacher's instructional time should be spent discussing multiplication with numbers less than one and multiplication larger than one. In each case the primary task is to show that some multiplication does not "make bigger."

EXTENSION: As a way of closing this lesson the teacher might assign the following questions for homework: How long would it take the Toads to run the 78 mile Tecate-Ensenada race? If Leona only wants two-thirds of this recipe, how many ounces of tomato paste will she need? How much would the buyer save on this CD purchase? How many marbles are not cat's eyes?

ANSWERS:
1 a) yes b) will vary c) 78 miles d) 13 miles
2 a) more b) will vary c) 9 ozs
3 a) less b) 2 ozs
4 a) less b will vary c) $309.47
5 a) yes b) will vary c) 555 marbles
6 a) no b) will vary c) 444 marbles
7 a) will vary b) will vary

EXTENSION ANSWERS: 5.42 hours, 6 ounces, $18.01, and 111 marbles

PARTS OR GROUPS

1. Every year the Jamul Toads run the Tecate-Ensenada foot race. Each member of the six-person team runs a 13 mile leg. a) If each runner runs at a 5 minute per mile pace, would the team run farther than 13 miles? _____ b) Write an equation that you would use to determine how far the would team run. _________ c) How far would the team run? ______ d) How far would each person run? __________miles

2. Leona loves to make a special pasta dish. Her recipe calls for 2 cans of tomato paste. a) If each can has a capacity of 4.5 ounces,will she use more or less than 4.5 ounces? __________ b) Write an equation you would use to determine how much of this ingredient she will use. __________ c) How much of this ingredient will she use? __________ounces

3. Bill has a 6-ounce chocolate candy bar. He can eat only $\frac{1}{3}$ of it. a) Will he eat more or less than 6 ounces? __________ b) How many ounces will he eat? _____

4 a) If Tau Electronics offers a 5.5% discount on any purchase over $200.00, will you pay more or less than the regular selling price for a CD player that normally sells for $327.48? __________ b) Write an equation you would use to determine how much you will pay for that CD player. __________ c) How much will you pay for the CD player? $____________

5 a) If there are 37 bags of marbles with 15 marbles in each bag, and each bag is worth $2.21, are there more than 15 marbles altogether? _________ b) Write an equation you would use to determine how many marbles there are in all 37 bags. _________ c) How many marbles are there in all 37 bags? _________marbles

6 a) If $\frac{4}{5}$ of the marbles in problem number 5 are cat's eyes, does the number of cat's eye marbles total more than 555? __________ b) Write an equation to determine how many cat's eye marbles there are. ______ c) How many cat's eye marbles are there exactly?__________marbles

7. Write a statement that summarizes the effects of:

a) Multiplying by a positive number less than one.__________

____

____

b) Multiplying by a number larger than one._______________

____

____

THE PAY-OFF

FOCUS: Application Activity for Multiplication

- Part of amount

PURPOSE: The student will...

- Estimate on a drawing part of an amount for given percents and fractions;
- Practice applying part-of-amount multiplication word problems;
- Interpret results; and
- Share and discuss methods and explanations.

STUDENT BACKGROUND: Students should possess percent and fraction number sense.

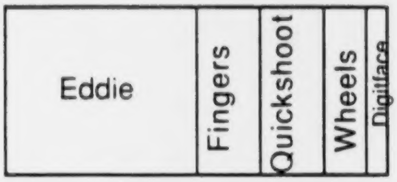
TEACHER BACKGROUND: Students may believe that in order to find part of an amount they need to divide ("multiplication makes bigger, division makes smaller"). This misconception needs to be dispelled, and the notion that multiplication will make smaller when multiplying with fractions and decimals less than one needs to be instilled.

MATERIALS: THE PAY-OFF worksheet

LESSON DEVELOPMENT: This exercise should be done in small groups. Students may choose to use percent to represent each share (converting fractions to decimals) or use percent and fractions as given. Groups should share methods and explanations with the class. Stress that "part-of-amount" word problem involve multiplication. All money figures do not come out in whole cents; you may need to remind students to round off to the nearest cent.

ANSWERS:

1.


Eddie
Fingers
Quickshoot
Wheels
Digitface

2. Slick Eddie- $175,000
   Quickshoot- $70,000
   Fingers- $70,000
   Wheels- $23,333.33
   Digitface- $11,666.67

EXTENSION: Wheels- $70,000
Digitface- $35,000

THE PAY-OFF

Tricky Tracey was working on a new case. Slick Eddie and his boys, notoriously known as the Safecrack Gang, were spotted in Smalltown. Eddie had come up with a way of breaking into the National Bank on Main Street, and his pals pulled it off. They had heard that $350,000 was stashed in the safe, and they stole every last penny. Eddie has always been the brains behind their operation and demanded half of the loot. Quickshoot and Fingers each get 20% of the total take. Slick Eddie is slick, but not slick enough to figure out the math, so he hires you, Digitface. Eddie promises you $\frac{1}{3}$ and Wheels $\frac{2}{3}$ of the remaining cash.

Answer the following questions, Show all work.

1. The figure represents the $350,000.00. Show on the drawing about how much of the whole amount Eddie, Quickshoot, Fingers, Wheels, and yourself, Digitface, will get.
Label each part with the name of the person.

$350,000

2. Now, Digitface, calculate everyone's share of the $350,000.00.
   Eddie:
   Quickshoot:
   Fingers:
   Wheels:
   Digitface:

Just as Eddie is about to dole out the cash, Tricky Tracey bursts into the hideout, and surrounds the place with the men in blue (policemen). Once again Eddie and his boys learn that crime doesn't pay. Tricky Tracey shakes your hand and thanks you for your undercover work. Case closed.

EXTENSION: Pretend Quickshoot gets "taken out" during the robbery by a security guard. Now, the remaining pot that you and Wheels share is larger. You still get $\frac{1}{3}$, Wheels $\frac{2}{3}$. How much does that come to? (Make a drawing)

HIGH SCHOOL ENROLLMENT

FOCUS: Meaning-Centered Lesson

- Percent

PURPOSE: The students will...

- Practice estimation of a percent by shading a drawing;
- Learn that when taking less than 100% of a number your answer is less than the original number;
- Learn that to find a percent of a number you multiply the number by the percent; and
- Gain experience with answers that don't fit reality (such as 2. people).

STUDENT BACKGROUND: Students should have previous experience with the meaning of percent, converting percents to fractions and to decimals, and with rounding off.

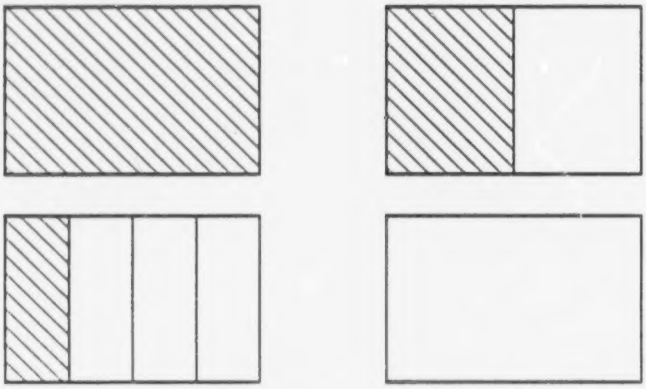
MATERIALS: HIGH SCHOOL ENROLLMENT worksheet

LESSON DEVELOPMENT: The worksheet may be done orally or handed out to the class as a whole, to small groups, or as independent work. In all cases, there should be a dialogue with the class on what the problem is asking and on the meaning of percent. The teacher should review as necessary how to convert percents to fractions and to decimals.

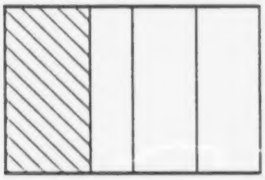
ANSWERS:

1. 2196
2. freshmen
3. number of freshmen
4. students or freshmen
5. $1=1$, $\frac{1}{2}=0.5$, $\frac{1}{4}=0.25$, $0=0$
6. $\frac{3}{10}=0.3$

7.



8.



9. 1 X 2196 = 2196,
   $\frac{1}{2}$ X 2196 or 0.5 X 2196 = 1098,
   $\frac{1}{4}$ X 2196 or 0.25 X 2196 = 549,
   0 X 2196 = 0
10. (pictures may vary but should resemble drawings in answer 7)
11. less, less, more, more
12. 30% X 2196 or $\frac{3}{10}$ X 2196 or 0.3 X 2196 = 659
13. approximately 659 freshmen
14. (read student explanations in order to understand their reasoning)
15. approximately 615 sophomores; 0.28 X 2196
16. approximately 505 juniors; 0.23 X 2196
17. approximately 417 seniors
    (100% - 30% - 28% -23%) X 2196 or
    2196 - (659 + 615 + 505)

HIGH SCHOOL ENROLLMENT

The expected enrollment for the high school you will be attending as a freshman is 2196 students. Your freshmen class is expected to be 30% of the total school enrollment. Let's find out how many students will be in the freshmen class.

1. How many students attend this high school?____________________
2. 30% represents which group of students?____________________
3. What will your answer represent?____________________
4. How will you label your answer?____________________
5. Knowing that percent means out of 100 or hundredths, write each percent as a simplified fraction and as a decimal.

100% = ________ = ________
50% = ________ = ________
25% = ________ = ________
0% = ________ = ________
fraction decimal

6. How would you write 30% as a fraction and as a decimal?

30% = ________ = ________

7. In each of the four boxes, shade in the given percent.

100%
50%
25%
0%

8. In the box below, shade in what you would estimate to be 30% of the box.

9. Knowing that 100% = 1, 50% = $\frac{1}{2}$ or 0.5, 25% = $\frac{1}{4}$ or 0.25, and 0% = 0, write a multiplication problem that represents each of the following:

100% of 2196 ________ X ________
50% of 2196 ________ X ________
25% of 2196 ________ X ________
0% of 2196 ________ X ________

10. Draw a picture to represent the answer to each of your multiplication problems in Problem 9. Label each picture.

11. Will your answer to the original problem be more or less than

2196 ? ___________
1098 ? ___________
549 ? ___________
0 ? ___________

12. What equation would you write to solve the original problem?

____

13. Solve the problem. ________

14. Does your answer seem reasonable? Why or why not? Explain .

Draw a picture and write an equation to solve problems 15 and 16.

15. 28% of the students at your future high school will be sophomores. How many students will be sophomores?

16. 23% of the students will be juniors. How many students will be juniors?

EXTENSION:

17. How many students will be seniors? There are two ways to get the answer. Can you find both solutions? Show your work for each solution.

FIND AREA AND VOLUME

FOCUS: Meaning-Centered Lesson for Multiplication

- Special cases: area and volume

PURPOSE: The student will...

- Know why to multiply when finding area, and
- Know why to multiply when finding volume.

STUDENT BACKGROUND: Vocabulary: dimension, length, width, quadrilateral, square, rectangle, rectangular solid, vertical, horizontal, height, diameter, circumference, congruent, parallel, radius, volume, area, π, vertex and cylinder.

TEACHER BACKGROUND: Students need to be less concerned with memorizing formulas and pay more attention to "developing an understanding of geometric objects and relationships," when solving problems dealing with geometric situations. The problems in this lesson enable the student to look at a concrete situation and see how it relates to multiplication, rather than unconsciously multiply because a formula indicates a particular operation.

MATERIALS: FIND AREA AND VOLUME worksheet, scissors, extra paper, 10 quarters, a metric ruler, and a calculator.

LESSON DEVELOPMENT: The important thing to develop in this lesson is that students should understand why they multiply to find area and volume.

The teacher's task when introducing AREA to students will be to show that it is similar to the idea of equal groups/amounts multiplication. In a figure, for example, that has a length of 5 cm and a width of 3 cm, it will be important for the teacher of this lesson to illustrate for the students that for each unit of length there is a group of 3 units of width. This group of 3 occurs 5 times. It is significant to note that this can also be instructed as a group of 5 (units of length) that occurs 3 times.

VOLUME should also be presented in a similar way. The teacher should begin to illustrate volume as multiple unit layers. If there is a volume problem that has a length of 5 cm, a width of 3 cm, and a height of 6 cm, then the teacher could begin by representing the area of one face as showing a layer. The area of the bottom (5 cm x 3 cm), for example, could be identified with the bottom layer. This layer is repeated 6 times. Be sure to note that a layer has thickness. In this case, the thickness is 1 cm. This is distinct from area, which has no thickness.

It should be clear to the students that a layer could be length by width, width by height, or height by length. With any of these options, a "unit layer" can be shown to occur a certain number of times in order to find volume.

EXTENSION: Have students draw a parallelogram that has sides that are 2.5 centimeters, a base that is 3 centimeters, and a height of 2 centimeters. See if they can create grids that show the meaning of BASE x HEIGHT. Some discussion of the extra triangular areas will be necessary.

One good idea is to have the students cut out the parallelogram. Then have them cut away one right triangle from either the lower right vertex, or the upper left vertex. Paste or lay it on the opposite side from where it was cut. It makes a rectangle with length and width equivalent to the base and height of the parallelogram. This might facilitate some discussion of the meaning of A = bh. Have them find the area.

Next, have your students observe a stack of ten quarters. Let a volunteer measure (in millimeters) the thickness and diameter of one of the quarters. Have the class try to draw this stack as a right cylinder, made of ten layers of quarters. Then discuss this particular volume in terms of multiple layers of quarters. Have them find the volume.

ANSWERS:

1a) square, b) 5, c) 5, d) 5, e) see drawings, f) 5 x 5,
g) 5 x 5 = A, and h) 25 cm^2

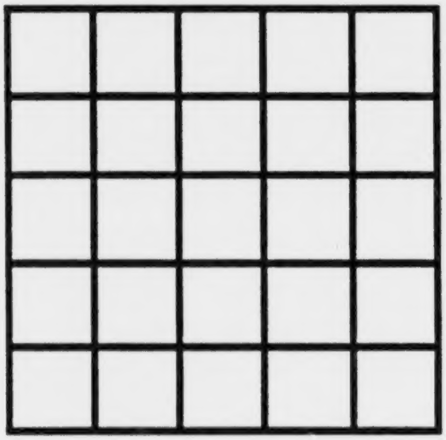
2a) rectangle, b) 7, c) 3, d) 3, e) see drawings, f) 7 x 3
g) 7 x 3 = A, and h) 21 cm^2

3) Answers will vary, but should include the idea of l x w = area, and areas as the numbers of unit square regions needed to cover the rectangle.

4 a) base x height, b) 1, c) 2, d) 3, e) (4 x 2) x 3, and
f) Answers will vary, but should include the idea of:
(base x height) x width = volume, and volume as the number of unit cubes needed to fill or match the cube or solid.

FIND AREA AND VOLUME

1. Consider all dimensions to be measured in centimeters. Answer questions about this shape:



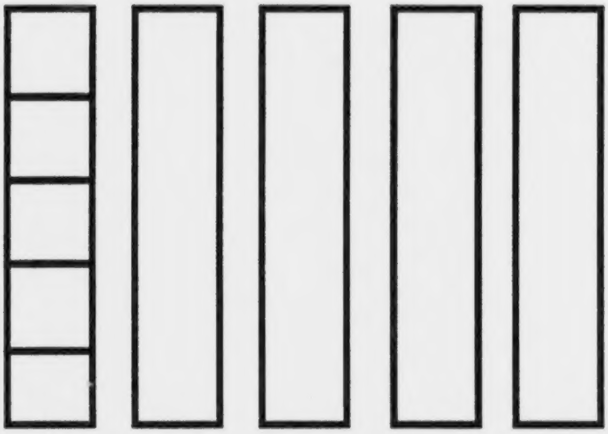
a) A quadrilateral with 2 pairs of parallel sides, four congruent angles, and four equal sides is a ___________

b) How wide is this figure? _____centimeters

c) How high is this figure? _____centimeters

d) How many vertical columns (groups made of 5 unit squares each) are in this figure? _____

e) Illustrate your answer for Problem 1 d) by completing the figures below:

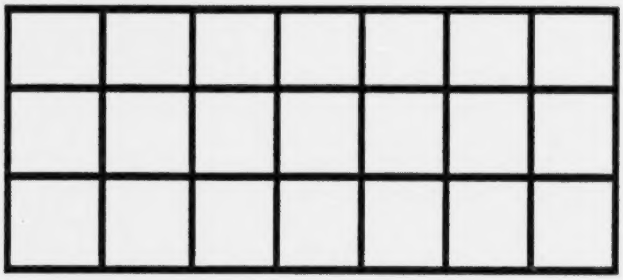


f) Write a phrase with numbers that means 5 groups of 5 _____

g) Write an equation for finding the area of Problem 1 ________

h) Count and record the squares of the 5 vertical columns _____

2. Assume the marks are one centimeter apart. Answer questions about the following figure:



a) Any quadrilateral with four 90° angles, two pairs of parallel sides, and two pairs of congruent sides is a __________.

b) How wide is this figure? _____ centimeters

c) How high is this figure? _____ centimeters

d) How many horizontal rows (groups made of 7 unit squares each) are in this rectangle? _____

e) In the space provided below Illustrate your answer for Problem 2 d) by drawing your answer. See Problem 1. e) for a hint.

f) Write a phrase with numbers that means three groups of 7 ____

g) Write an equation for finding the area of Problem 2 _________

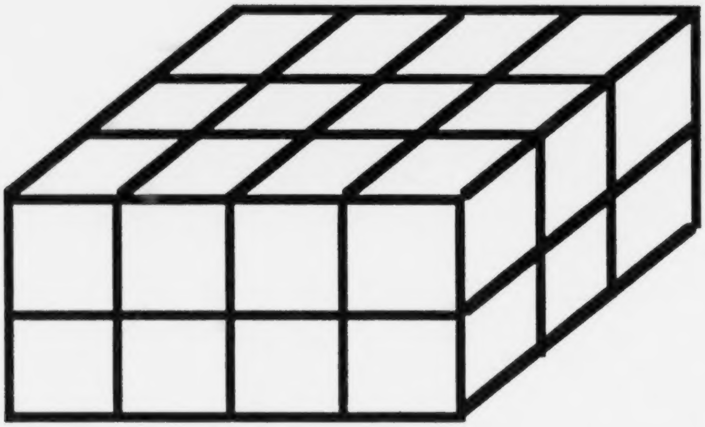
h) Count and record the unit squares in Problem 2 _____

3. Consider Problems 1 and 2. Express what it means to find the area of a square or rectangle. ___________________________

____

____

4) Consider all dimensions to be measured in meters. Answer questions about the following figure:



a) Without counting, how can you determine how many cubes there are in the front layer of this rectangular solid? Show your answer as a phrase with numbers: _______

b) How thick is each cube? _____meter

c) How many more layers are exactly like the front layer? _____

d) How many 4 x 2 layers are in this figure? _____

e) Write a math expression to indicate how to determine the number of cubes in this rectangular solid: ___________

f) Express what it means to find the volume of a rectangular solid:

____

____

____

WHAT'S IN AN AREA?

FOCUS: Meaning-Centered Lesson for Multiplication
- Area

PURPOSE: The students will...
- Learn the conceptual definition of area;
- Derive the formula for the area of a rectangle;
- Realize that multiplication is used to find area regardless of the kind of number being used;
- Use appropriate units of measure; and
- Relate the concept of area to objects in real life.

STUDENT BACKGROUND: Students may have little or no understanding of the concept of area. They should know the following vocabulary: square, length, width, rectangle, formula, diagonal, vertical, and horizontal. Students should also have fraction and decimal sense.

TEACHER BACKGROUND: Students need to understand the meaning of area, rather than just use a formula to get an answer. Understanding a concept helps them to apply that concept.

MATERIALS: WHAT'S IN AN AREA? worksheet, ruler, scissors.

LESSON DEVELOPMENT: Students are to work in small groups.

Discuss with the class the meaning of length (which tells how many squares in one row) and width (which tells how many rows).

Use the square in Part 1 and the rectangle in Part 3 in a class discussion about how finding area relates to multiplying by groups, i.e., you have a length of 5 inches occurring 3 times. Investigate with the class how this relationship also applies to the square in Part 2.

Encourage students to use appropriate language (length, width, square inches, area, etc.) when giving written and oral explanations for the worksheets and for the extension. If the extension is assigned, have students share and compare their answers and the way they solved each problem.

ANSWERS:

1. - - -
2. - - -
3. 4 square inches
4. Answers will vary
5. Multiplication; The area of a square can be found by multiplying the length by the width.
6. - - -
7. - - -
8. 9 square inches
9. - - -
10. Yes
11. 3 square inches
12. $\frac{1}{4}$ or .25 of a square inch
13. $9 + 3 + \frac{1}{4} = 12\frac{1}{4}$ inches
14. $3\frac{1}{2} \times 3\frac{1}{2} = 12\frac{1}{4}$ square inches
15. - - -
16. length x width
17. Yes
18. 15 square inches
19. The area of a rectangle can be figured out the same way as finding the area of a square; multiplying the length by the width.
20. $7\frac{1}{2}$ square inches
21. 2 square inches; $4 \times \frac{1}{2}$
22. No; square feet or yards
23. Answers will vary, (city, country, etc.)

WHAT'S IN AN AREA?

PART 1

1. On another piece of paper, use your ruler to draw a square that is 1 inch long on each side. Label it, "One Square Inch".
2. Draw a square that is 2 inches long on each side.
3. Cut out your 1 inch square from Problem 1, and figure out how many of them will fit inside your square in Problem 2. Be sure to label your answer as square inches. Your answer is called the area of the square.
4. Have each person in your group draw a square, each a different size, with the length of a side in whole inches. As a group, figure out how many square inches fit inside each of your big squares.
5. Can you and your group figure out how many one inch squares will fit inside another bigger square by using a mathematical operation? Explain why that operation works:

____

____

PART 2

6. Use your ruler to draw a square that is 3 $\frac{1}{2}$ inches long on each side.
7. Using your ruler, draw lines through your square at 1 inch intervals, vertically and horizontally.
8. How many complete one inch squares (not counting parts of a square inch) do you see in this 3 $\frac{1}{2}$ square? ________________
9. Cut out the remaining parts that are not complete square inches.
10. Will any of these parts fit together to make 1 square inch? (Equal in size to the square inch you cut out in Problem 1)

    ____

11. How many square inches do these extra part make?__________
12. You should have one more piece that is smaller than 1 square inch. Using your cut-out square inch, can you figure out what part of a square inch this last extra piece is? (You may express your answer as a fraction or decimal).

    ____

13. Use your answers from Problems 8, 11, and 12 and write an addition problem. Solve. ______________________________

14. Use the same mathematical operation you used in Problem 5 for your $3\frac{1}{2}$ inch square. ____________

15. Your answers to Problems 13 and 14 should be the same. If they are not, work Problems 6 through 14 again with your group to find the error.

PART 3

16. In general, can you find one formula to find the area of any square, no matter how long the sides are?

17. Do you think this formula would also work for rectangles?

18. Draw a rectangle 5 inches long and 3 inches wide. Determine the area of this rectangle by counting the number of square inches that fit inside it AND by using your formula.____________

19. What did you discover? Explain why the formula works:

____

____

____

20. Draw a diagonal line through your rectangle in Problem 18. What do you think the area of each triangle is?______________

21. Find the area of a rectangle 4 inches long and $\frac{1}{2}$ inch wide. Explain how you got your answer. ____________________________

____

____

____

____

22. If you were to find the area of the classroom floor, would it be appropriate to measure it in square inches? What unit of measure for each square unit would be appropriate?

23. What might you be measuring if you used a square mile as your unit of measure?

EXTENSION:

Find the area of the following. Label your answer with the appropriate square units.

1. this piece of paper
2. the cover of your math book
3. a postage stamp
4. your desk top
5. the classroom door
6. the classroom floor
7. a magazine cover
8. a calculator
9. a ruler
10. something at home
11. Estimate the area of the bottom of your foot, (trace your foot or make a footprint.)

AT ANY RATE

FOCUS: Application Lesson for Multiplication
- Rate

PURPOSE: The students will...
- Solve rate problems, and
- Write explanations to show comprehension.

STUDENT BACKGROUND: In order to solve these problems students should be able to make some simple time conversions.

Students should also be aware of real-life rounding. For example, 0.2 blocks is meaningless in terms of some action with whole blocks. So, an answer like 4.08 blocks perhaps should be rounded to 5 blocks, and not the usual 4.

Money needs to be rounded to the nearest cent.

MATERIALS: AT ANY RATE worksheet, and calculator.

LESSON DEVELOPMENT: Many students may find that this is a difficult assignment. Most of the situations involve multi-step strategies. Therefore, it may be useful for the teacher to assign this lesson to small groups for collaboration.

EXTENSION: 1. g) Discover the minimum number of moves it would take for Thanh to win a stalemate with Melinda if they played at the constant rate stipulated in Problem 1 and Thanh made the last move. State your answer and explain your thinking: ______________________________

____

____

____

____

____

____

ANSWERS:

1a) Thanh
b) Thanh
c) Thanh
d) Thanh's Time = 35 sec/move (22 moves)
Melinda's Time = 37 sec/move (21 moves)
e) T = 770 seconds < M = 777 seconds therefore Thanh would win, if a stalemate were declared at this point.
f) Answers will vary, but should focus around the notion that Thanh's rate is quicker than Melinda's and that therefore when an equal number of moves are made, Thanh's rate will yield him less time compared to Melinda.

EXTENSION 1. g) Answers will vary, but Thanh must play 19 moves to win a stalemate with his last move. Students might mention the difference between RATE x #moves decreases by a common difference of -2 (i.e. 2 less each time):

[T = 35(2)] - [M = 37(1)] = 33 seconds more for Thanh
[T = 35(3)] - [M = 37(2)] = 31 seconds more for Thanh
[T = 35(4)] - [M = 37(3)] = 29 seconds more for Thanh
etc.
[T = 35(18)] - [M = 37(17)] = 1 second more for Thanh
[T = 35(19)] - [M = 37(18)] = 1 second less time for Thanh. He wins!

2a) 500,000 = [9(60) + 40](B) ((answers may vary))
b) 862.06896551724 = B rounded to 863 blocks

3a) $85,000/year = [7 hours][52 weeks][$/hour]
Answers may vary
b) $233.51648351648/hour rounded to $233.52/hr
c) 28 hours/month
d) $85,000 ÷ 12 = $/mo
e) $/month = $7083.33/month

4a) $176,000 + 3d = $325,000
d = $49,666.67
b) 1990 selling price = $176,000 + $49,666.67
c) 2000 selling price = {2000 - 1989}($49,666.67) + $176,000
2000 selling price = $722,333.37
d) probably
e) Answers will vary but should include some comparison of 1989 selling price vs 2000 selling price.

AT ANY RATE

1. At Zorba Middle School Melinda and Thanh were playing a very close game of chess. The tournament director stipulated that if any game ended in a stalemate, the player with less total time would win. Melinda averaged 37 seconds per move for 21 moves, and Thanh averaged 35 seconds for each of his 22 moves.

a) Who started the game? __________
b) Who made the last move? __________
c) Who made the most moves? __________
d) Write an equation for the time played by each player.
Thanh ________________ Melinda ________________
e) Who played the least amount of time so far? __________
f) Explain why Thanh would always win any stalemate ending with Melinda's move, in this game . ______________________________
____
____
____
____

2. Pink Floyd planned to perform at the Berlin Wall to celebrate its recent fall. Architecture for the set was going to be designed in such a way that a prefabricated wall made of 500,000 styrofoam blocks could be tumbled down, at a constant rate (block by block), in less than 9 minutes and 40 seconds. If the show is going to have the intended impact, how many blocks must be tumbled over each second?

a) Write an equation for this problem: _______________
b) How many blocks per second will be tumbled over? __________

3. Robert Allen, author of Nothing Down, makes a claim that if a person is willing to invest seven hours per week (5 days) they could earn up to $85,000 in a year's time. If this is true...

a) Write an equation for how much money could be earned per hour.
____
b) How much money could be earned for each hour worked? ______
c) How many hours would a person work each month? __________
d) Write an equation for how much money could be earned in a month. _______________
e) How much money could be earned in one month? __________

4. In 1989 the average cost of a house in San Diego Country Estates was \$176,000. By 1992 the average cost is expected to be \$325,000.

a) What was the average dollar rate of increase for one year? \$____

b) Write an equation to determine the average selling price of a house in 1990.

____

c) What will be the cost of a house by the year 2000 at this rate of increase? __________

d) If you were a speculator, would buying a house in this area be a good investment? __________

e) Explain your answer for Problem 4d: ______________________

____

____

PULSE RATE

FOCUS: Meaning-Centered/Application Activity

- Rate
- Additive comparison
- Multiplicative comparison

PURPOSE: The students will . . .

- Become familiar with the concept of rate through direct experiences;
- Develop an understanding of rate through multiple representations of rates in tables and graphs;
- Learn of the distinction between additive and multiplicative comparison by contrasting, analyzing, and writing problems of each type;
- Develop some appreciation for the relative sizes of large numbers; and
- Learn to reverse the direction of comparison in the two types of comparison problems and become aware of the different effects in the two cases.

STUDENT BACKGROUND: Students need to use decimal and large-number number sense. They are expected to be familiar with comparison subtraction and part-of-a-group-or-amount multiplication. Relationships between addition and subtraction and between multiplication and division should be familiar. Students need to be able to construct bar graphs and line graphs, to read and write large numbers, and to estimate.

TEACHER BACKGROUND: Rate is a complex concept to be approached here through use of students' active involvement in finding and using their own pulse rates, multiple representations for rates (e.g., table, graph), and extensive discussion. The contrast between additive and multiplicative comparison is important for the development of proportional reasoning ability.

The difference in effect of reversing the direction of comparison in the two cases might contribute to greater awareness of the distinctions present in additive and multiplicative situations. Such awareness might help students see that the relationship in proportions is multiplicative rather than additive as many now believe.

MATERIALS: Ways for students to time 15 seconds accurately in Parts 1 and 2.

Part 1: PULSE RATE worksheet
Part 2: COMPARISON OF PULSE RATES worksheet
Part 3: ANIMAL RATES worksheet

LESSON DEVELOPMENT: This investigation is designed in three parts, each with a new concept. Part 1 introduces the concept of rate in a concrete way. Part 2 contrasts comparisons made using additive relationships with multiplicative comparisons. Part 3 extends these ideas and shows the effects of reversing the order of comparison in the two types of situations.

Part 1: Pulse Rate

Each student takes his own pulse for 15 seconds then determines pulse rates in beats per minute. Determine the range of pulse rates and construct charts on which to record the pulse rates of the students. Separate charts are constructed for boys and girls because the average pulse rate of females (78-82) is higher than that for males (70-72). Through discussion, determine a single scale appropriate for students to use in making bar graphs of the class frequencies of the rates for both genders.

Students are asked to estimate in this lesson, once using a graph and once using computation with large numbers.

The comparison in Problem 4 is intended as an additive one in which the students may review use of greater than/less than. Multiplicative comparisons will first appear in Part 2.

Part 2: Comparing Pulse Rates (Additively and multiplicatively) Students will do jumping jacks in class to get an active heart rate, or arrangements can be made with the physical education teacher to have the students do aerobic exercise and determine active heart rates during PE class. Students will make bar graphs using the data on active heart rates. "Heart rates may increase to more than 200 beats per minute during violent exercise, and decrease to 12 during extreme cold weather."[1]

Students need access to the Part 1 worksheet during this lesson in which they compare various rates additively (how much greater than) and multiplicatively (how many times as great as).

The relationships between addition and subtraction and between multiplication and division are important. Additive comparison requires subtraction to find the difference; multiplicative comparison requires division to find the number of times as great.

Be sure to summarize: There are two ways to compare quantities:
How much greater (less) than (usually requires subtraction).
How many times as many (much) as (usually requires division).

Part 3: Rates in Animals (Effects of reversing order of comparisons)

Students continue to contrast the two types of comparison using animal heart and breathing rates. Students are expected to consolidate the information, to generalize it, and finally to use the two types of comparisons in writing problems of their own.

The data provided on animal heart and breathing rates are approximations often resulting in easy mental computations. When this is not the case, estimation and mental computation, rather than exact answers, can be encouraged (for example, Problem 1a).

Problem 2 should be answered and discussed in class before students are expected to answer Problem 3.

That the difference remains the same when the comparison is reversed in the additive situation may be reinforced by doing the reversal of order in earlier additive problems (from Parts 1 and 2) after encountering the idea in this lesson.

The difference in the behavior of the additive and multiplicative rates when the directions of the comparisons are reversed may be difficult, so Problem 6 might be considered an extension problem. Use of the expressions **comparison factor** and **comparison term** (or difference) might be used if it would seem to illuminate the contrast.

The class might now make a chart of the terms encountered in these lessons and the operations they use in each situation as part of the process of summarizing the material.

ANSWERS:

Part 1 PULSE RATE

1. Answers vary. Average is about 70 per minute; mine (female, no exercise) is 84; my husband's (exercises) is 60.

2. Average female pulse rates are greater than male. This may or may not show up in the class data.

3. Answers vary. **Using 70 beats per minute**, the answers would be:
Table: 140, 280, 420, 560, 700, 840, 980.

Using 70 beats per minute,

a) About 945 beats in 13.5 min.

b) 100,800 beats in one day; 70 x 60 x 24

c) About 10 days after birth since there are 10 hundred-thousands in 1 million and the heart beats about 1 hundred thousand beats per day.

d) Will vary.

One possible way: 1000 million = 1 billion;
1000 x 10 days = 10,000 days;
3 years = 1000 days, so
≈30 years old for billionth beat.

e) About 27 years
1,000,000,000 ÷ 100,800 = 9921 days
9921 ÷ 365 = 27.1 years

f) Answers vary.

4. Answers vary.

c. The **average** heart rate is faster than the **resting** heart rate and slower than the **active** rate.

Part 2 COMPARING PULSE RATES

1.

a) Active pulse rate graphs have higher frequencies at higher rates.

b) Answers vary;
Active rate - Resting rate = Difference

c) Answers vary.
6 x Average rate - Average rate = Difference;
6 times as many.
Beats in 6 minutes = 6 x Beats per minute

d) Answers vary; use subtraction.
Multiplication/division
Comparison in Part b is like the first comparison in Part d.

2.

a) Answers vary. My rate - Sandy's rate (43) = Difference

b) Yes. Rate means beats per minute in this case, so the difference in rates is the difference in number of beats per minute for Sandy's heart and mine.

c) 60 x (answer in Part a)

d) About 2; My rate ÷ Sandy's rate ≈ 2 **or** 2 x Sandy's rate = My rate

e) About 2. Logic or $\dfrac{\text{My rate x 60}}{\text{Sandy's rate x 60}}$

= My beats per hour ÷ Sandy's beats/hour

≈ 2

f) The answers in Parts a) and c) are **different**.
The answers in Parts d) and e) are **the same**.

g) Each minute my heart beats (about 30) more beats than Sandy's, so in an hour it beats 60 x (about 30) more times. The number of **times as many mine** beats is the same during any period of time.

3. Answers vary.

Part 3 ANIMAL RATES

1. (Encourage the use of estimation and mental computation in this lesson)

a) 10

b) Answers vary. Same answer.

c) 20; !00 ÷ 5 = n or 100 = 5n

2. Answers vary. Range might be 90 - 100, based on its size.

3. Size, activity level. Answers vary.

4. 210

5. Explanations should include the distinction between finding "how many more or less than" in additive comparison and finding "how many times as many as" in the multiplicative comparison.

Examples vary.

The "how many more than" comparison seems more natural in this case.

6. 50

a)

b) 1/50

c) If one rate is **n** times as great as another, the second will be one-**n**th as great as the first.

d) Whale's rate is about 1/10 the human's rate.

7. Answers vary.

References

Gillen, D. (1989). Hummingbird how-tos. *Zoonooz*, **62** (7), 8-11.

Hart, K. M. (1984). *Ratio: Children's strategies and errors.* Windsor, Berkshire: NFER-NELSON.

McFarlan, D., (ed.). (1990). *Guinness Book of World Records.* New York: Bantam.

PULSE RATE

Part 1

1. Determine your pulse rate in the following way.

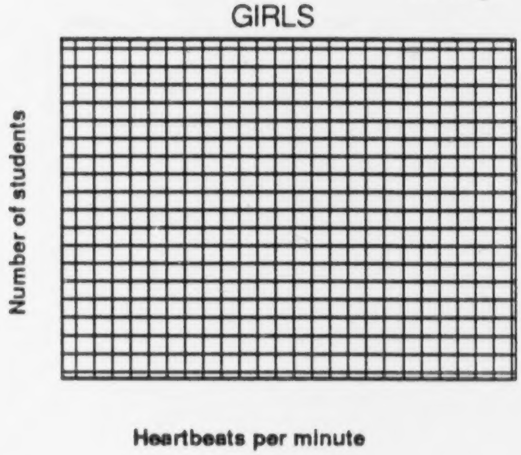
a) How many times does your heart beat in 15 seconds? _____

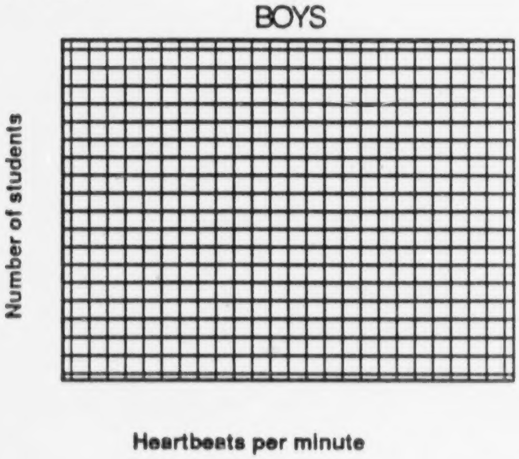
b) How many times would your heart beat in **one** minute beating at the same pace? _______

c) Therefore, your pulse rate right now is _______ beats **per** minute or _______ $\frac{\text{beats}}{\text{minute}}$. Use this as your **average** pulse rate.

2. Using the data collected in class, make one bar graph for the pulse rates of the girls in your class and one for the rates of the boys.

Average Pulse Rate



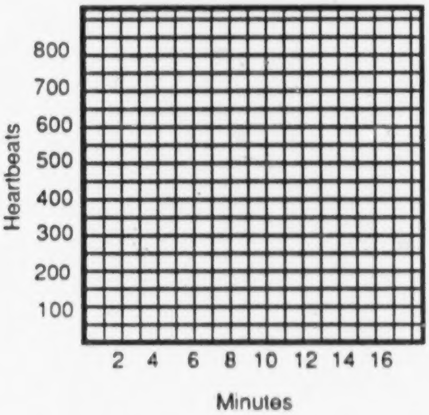


Is there any noticeable difference between the two graphs? _______
If so, describe the difference. ______________________________
____
____

3. Complete the table and then construct a line graph to show the number of times **your** heart beats over several minutes.

Heartbeats

Minutes	Heartbeats
2	
4	
6	
8	
10	
12	
14	



Your Heartbeats

a) Use your graph to estimate the number of times your heart beats in 13.5 minutes. ____________
What equation could you use to get this information?

At this rate,

b) How many times does your heart beat in one day?

____

Show how you reached your answer.

c) Starting from birth, when will/did your heart beat its one millionth beat? _________
How do you know? ____
____
____

d) Estimate the age at which your heart will beat for the one billionth time. _________________________
How did you arrive at your estimate? ____________________________
____
____

e) Calculate the age you will be when your heart beats for the one billionth time. ______________________________
Show how you reached your answer.

Was your estimate reasonable? _______________

f) Would you have guessed that a billion beats would take this much longer to reach than a million? _______

4. Homework:

a) Find your heart rate when you first awaken in the morning (your **resting** heart rate): ______ $\frac{\text{beats}}{\text{minute}}$

b) Compare this rates and your **average** rate (the rate you found in class).

____

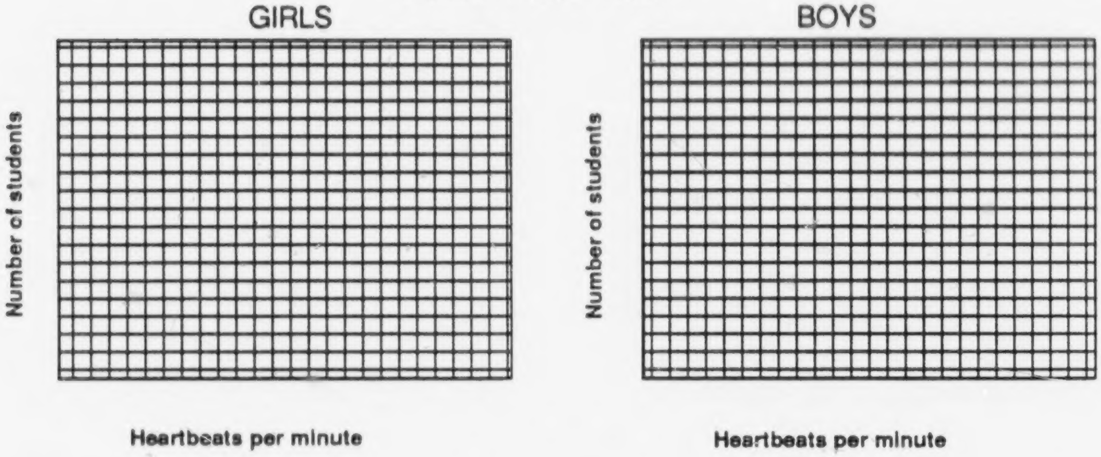
____

COMPARING PULSE RATES

Find your **active** heart rate as instructed by your teacher: ______ $\frac{\text{beats}}{\text{minute}}$

Make one bar graph to show the active pulse rates of the girls in your class and one to show the active pulse rates of the boys.

Active Pulse Rate



1. There is more than one way to compare numbers.

a) What can you say about the **average**-pulse-rate bar graphs you drew earlier and these **active**-pulse-rate bar graphs? ________________

____

____

b) How much is the difference between your resting pulse rate and your active pulse rate? __________

What equation would you write to express this difference?

c) Refer to your table in Problem 3 (in Part 1).

Compare the number of heart beats in six minutes with the number in one minute in two ways:

Your heart beats ______ **more beats** in 6 minutes than in 1 minute.

What equation would you write to express this relationship?

Your heart beats ______ **times as many beats** in 6 minutes as it beats in 1 minute.

What equation would you write to express this relationship?

d) Contrast the two comparisons you made in Part c):
In 6 minutes, the number of beats is ______ beats **greater than** the number of beats in 1 minute. To compare this way, what operation do you use? ____________

In 6 minutes, your heart beats __ **times as many** beats as it does in one minute. To compare this way, what operation do you use?
____

Is the comparison in Part b) like the first or second comparison here?
____

2. Let us practice the two ways of comparing quantities. Sandy, a long-distance runner, has an average pulse rate of $43 \frac{\text{beats}}{\text{minute}}$. Since Sandy is in very good physical condition, each beat of her heart is stronger and pumps more blood than one beat in a person with a typical heart rate.

a) How much **greater** is your heart rate **than** Sandy's? __________
What equation would you write to express this comparison?

b) Is this the same as asking how **many more** beats your heart beats in one minute **than** hers does? _____
Explain ____
____
____

c) Your heart beats _____ beats **more than** Sandy's in one hour.

d) Your heart beats about ____ **times as many as** Sandy's in one minute.
What equation would you write to express this comparison?

e) Your heart beats about ____ **times as many as** Sandy's in one hour.
How did you find the answer? _______________________________
____
____

f) Are the answers to Parts a) and c) the same or different? ________

Are the answers to Parts d) and e) the same or different? ________

g) Explain the reasons for your answers in Part f).

____

____

3. According to the *Guinness Book of World Records*, "When Charles Thompson of Cwmbran, Gwent, Wales, was admitted to a hospital for hip replacement surgery on Aug. 16, 1987, he was found to have a record resting pulse of 28."

Using this information, write two word problems for your classmates to solve. At least one of the problems should require one of the two kinds of comparison you did in this lesson.

ANIMAL RATES

1. The table gives the average heart rates for some animals.

Animal Heart Rates

	Beats per min.
Whale	9
Gopher	100
Woodchuck	?
Hummingbird	1,260

Use the table to find the information missing in the following:

a) The average heart rate of a human (70 beats per minute) is about ________ times as great as the pulse rate of a whale.

b) A gopher's heart beats ______ $\frac{\text{beats}}{\text{minute}}$ **more than** yours beats.

Your heart beats _______ $\frac{\text{beats}}{\text{minute}}$ **fewer than** a gopher's.

Notice that when you change the direction of the comparison in this kind of comparison situation, the size of the difference is the same. Will this always be true in this kind of comparison? ______

Explain ____

____

c) If a gopher's **average** heart rate is 5 times its heart rate during hibernation, what is its hibernating rate in beats per minute? ____
What equation would you write to find the answer?

2. What factors seem to determine an animal's heart rate? ______

____

Explain why this might be the case. __________________________

____

____

3. A woodchuck's heart rate is "only a few beats per minute"[1] during hibernation. Estimate its normal heart rate. _____________
How did you reach your estimate? ____________________________

____

____

4. The woodchuck breathes only ten times each hour while hibernating. An active woodchuck breathes 2100 times per hour.[2] Its active breathing rate is _____ times as great as its breathing rate during hibernation.

5. Explain the difference in the two ways you have used to compare in this lesson. Give other examples of uses for each way.

____

____

____

____

____

Which of these two ways to compare is useful in explaining the differences in the graphs for the boys' pulse rates and the girls', if a difference was found? ____________________________________

6. If instead of breathing 200 breaths per minute (average rate, **a**), an animal takes 4 breaths per minute during hibernation, its average rate (**a**) is ____ **times as great as** its hibernation rate, (**h**).

a) Explain: ____

____

b) You could compare these rates in the **other direction**:
The animal's hibernation rate (**h**) is _________ **as great as** its average rate (**a**).
(The answer is a fraction. If the rates were the same, the hibernation rate would be **just as great** (1 x **h** = **a**); however, since the hibernation rate is smaller, the **comparison factor** will be less than 1.)

c) Explain the relationship between these two ways to compare the same two rates.

____

____

____

____

d) Reverse the direction of the comparison in Problem 1a). Write the new problem and its solution.

____

____

____

7. The ruby throated hummingbird beats its wings at the incredibly rapid speed of fifty to seventy times a **second**. It has a body temperature of 111°F. Its heart rate is 1,260 beats per minute.

If a 170 pound man expended energy at the rate of a hummingbird, he would have to eat 285 pounds of hamburger or twice his weight in potatoes each day to maintain his weight. He would have to evaporate 100 pounds of perspiration per hour to keep his skin temperature below the boiling point of water.

The average hummingbird weighs less than a penny. Its newborn are the size of bumblebees, and its nest is the size of a walnut. The hummingbird is the only bird that can fly backward.[3, 4, 5]

Using this information along with other information in this investigation (or other information you find on your own), write four problems for your classmates to solve. Include problems requiring both types of comparison you have used in this investigation. For one of the comparison problems, write the reverse-order comparison problem (in addition to the four problems).

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1 Gillen, D. (1989). Hummingbird how-tos. *Zoonooz*, 62 (7), Page 11.
2 *Isaac Asimov's Book of Facts*, Page 39.
3 *Isaac Asimov's Book of Facts*, Page 118.
4. *Fascinating Facts*, Page 22.
5. *Zoonooz*

CAN A MARBLE BREAK THE SPEED LIMIT?

FOCUS: Application Activity for Multiplication and Division
- Rate (speed)

PURPOSE: The student will . . .
- Collect distance and time data in feet and seconds, and
- Calculate the speed in miles per hour.

STUDENT BACKGROUND: The student should have had earlier work with d = rt types of problems and with conversions among units (e.g., seconds --> hours, feet --> miles).

MATERIALS: Yardstick or measuring tape, watch or clock with second hand, marble or large ball bearing.

LESSON DEVELOPMENT: Flip (i.e., flick or shoot) a marble the length of a chalk tray. [Alternatives include going outside, or using the floor, perhaps with lanes defined by tape, string, or books.]

Say, "I wonder how fast I can make this marble go?" Flip again, more forcefully. Ask, "How fast do you think it went that time?" It is to be hoped that estimates will cover a wide range, and you can pose the question of the lesson in this form:

> Can we make the marble break the speed limit in the school zone here at school?

Assign the problem to mixed-ability groups. Ask what data they need to answer the question. Either have each group collect its own data, or have two or three students generate data, with all the groups advising on procedure and using the "best" results.

Each group should prepare a report of its work; allow time for one or two groups to report their findings.

SOURCE: Idea from Carole Greenes' talk at the annual meeting of the Greater San Diego Council of Teachers of Mathematics, 1989.

ENDURANCE AND MATHEMATICS

FOCUS: Application Activity

- Rate problems

PURPOSE: The student will ...

- experience solving rate problems in real-life settings.

STUDENT BACKGROUND: Before students attempt these problems it is assumed they have sufficient skills working with fractions, decimals, and denominate numbers. It is also assumed that they have been exposed to D = rt problems.

TEACHER BACKGROUND: Remind students that decimal fractions do not equal time. For example: 1.5 ≠ 1 minute 5 seconds. Some discussion regarding converting time and decimals is necessary.

MATERIALS: ENDURANCE AND MATHEMATICS worksheet, and calculator.

LESSON DEVELOPMENT: It is suggested that this lesson begin with 5 minutes mental math rounding (i.e. 1 min 5 sec is about 1 min, and 2756 km is about 3000 kms.) This should be followed with 5 minutes discussing time conversions. Finally, talk about the endurance events to determine what the students already know and need to know. These problems are complex and, therefore, might be especially well suited for group collaboration.

Most students are going to find these numbers difficult. One help might be rounding off all numbers to the nearest hundredth.

Finally, all of these problems are multiple step situations. It might be worthwhile using one of these problems as a model to show students what kind of sequenced thinking is involved.

ANSWERS AND SOLUTION KEY

1. D = rt

55:55	2/60 = .03	31 = r(.92)
- :53	(55:03)/60 = .92 hrs	(31)/(.92) = r
55:02		33.7 km/hr = r

To win you must ride at 33.7 km/hr. A discussion of why 34 km/hr would be better might be effective.

2.

Pellon	You
30/60 = .5	5(10) = $(\text{time})_Y$
(5 + 2)(8.5) = $(\text{time})_P$	50 minutes = $(\text{time})_Y$
59.5 minutes = $(\text{time})_P$	

59.5 > 50 Therefore Pellon will not catch you within the next five miles. A good question is: Will Pellon catch you? If so, when? (In how many miles?)

3. 1:15 = 60 min + 15 min = 75 minutes Swim Time

4:26 = Run Time	(112)/(18) = Bike Time
4(60) + 26 = Run Time	6.2 hrs = Bike Time
240 + 26 = Run Time	6.2(60) = 372 mins = Bike Time
266 minutes = Run Time	

75 + 372 + 266 = 713 total Ironman competition minutes

15(60) = 900 minutes Ironman time limit

713 < 900 Therefore you will break the 15 hour time limit, and you will not be disqualified.

EXTENSION: Problem 4 could be used as an assignment for group collaboration. Groups could exchange their problems and evaluate each other's solutions. Group presentations work well with this kind of situation.

ENDURANCE AND MATHEMATICS

1. Imagine it is the last stage of the *TOUR DE FRANCE*. You are the last rider and you can win. After 2081 kilometers, you are only 52 seconds behind the leader. If only 31 kilometers remain in a final time trial event, how fast (km /h) must you ride if Finon (the leader) finished his last stage in 55 minutes and 55 seconds? Show your work. (Will 52 seconds faster win?)

Explain your thinking ______________________

2. You are leading the *Western States* 100 mile foot race through the Sierra Nevadas. After the river crossing at Rucky Chucky, you are growing weary. The next aid station is five miles ahead, all uphill and in the dark. Jan Pellon is challenging from two miles behind. If you can hold only a 10 minute per mile pace until you reach the next aid station at 87 miles, will Pellon catch you if she runs 8 minutes 30 seconds per mile? First try solving this with mental math and estimation, then use pencil, calculator and worksheet to solve it exactly. Show your work in the space provided below.

Explain your thinking for solving Problem 2:

3. You finally made it! You're in the *IRON MAN* . You are an excellent swimmer and can make it through the 2.2 mile swim in 1 hour and 15 minutes. You are a strong rider and can maintain 18 miles per hour for the 112 mile bike stage. When it comes to running, you'd rather watch. However, you are in this event and plan to finish. If your best Marathon (26.2 miles) is 4 hours 26 minutes, is it very likely you will break the 15 hour time limit? Try solving this first with mental math and estimation, then solve it on the worksheet. Show your work.

Explain your thinking:

4. Create your own problem that relates to any feat of endurance: (climbing Everest, any professional/Olympic sporting event, etc.)

LONG DISTANCE, PLEASE

FOCUS: Application Activity
- Rate

PURPOSE: The students will...
- Gain an understanding for how mathematics is used in real life situations involving rates;
- Learn to analyze a problem for the pertinent information, helping to lead
  to the appropriate operations; and
- Practice working with others and expressing their thinking.

STUDENT BACKGROUND: Students must be competent with rounding off decimals, and with conversions between minutes and hours.

TEACHER BACKGROUND: Research shows that real-life problems promote critical thinking, helping students to know what operations to use to solve problems.

MATERIALS: LONG DISTANCE, PLEASE worksheet

LESSON DEVELOPMENT: The lesson may be used as independent work, but preferably should be done in small groups.

The teacher may want to do the first part of Problem 1 with the class to show that all problems are multi-step problems.

Explain that the phone company charges a full minute for any time used that is less than a minute.

ANSWERS:
1. $16.89, $12.06, $6.64
2. 17 minutes, 24 minutes, 45 minutes
3. $18.49

EXTENSION:
1. If the lesson is done in group work, have each group defend its answer to Problem 3 to the rest of the class. Encourage the students to use language appropriate to the lesson (such as rate, per).

2. Ask students to use a phone book to find the needed information to calculate the cost of a 36-minute call from their house to the White House made on Monday, 7:30 am. Students must show all work and explain their solutions in writing.

3. Have students call the phone company and ask them how a customer is charged when your time crosses two or more rate periods. Ask any other questions that may come up during class discussions.

LONG DISTANCE, PLEASE

You and your family unexpectedly had to move to another city, and you are really missing those long talks you used to have with your best friend. You decide to call and catch up on the news back home, but you know your parents do not want a huge phone bill. You look in the phone book and figure out what the rates will be to call.

You discover the following rates apply to call your former hometown.

FULL WEEKDAY RATE 8:00 am-5:00 pm		EVENING RATE M-F 5:00 pm-11:00 pm	NIGHT RATE M-F 11:00 pm-8:00 am and all day on weekends
1st min.	$.37	$.26	$.15
Each Additional Minute	$.28	$.20	$.11

1. Suppose you decided you would hold your conversation to exactly one hour. Find what the cost would be if you called during each of the three different rate periods.

DAY RATE

EVENING RATE

NIGHT RATE

2. Your parents say they will not pay more than $5.00 per month for your long distance calls. You decide to use the $5.00 in one long call rather than several short calls. How long could you talk during each of the different rate periods?

DAY RATE

EVENING RATE

NIGHT RATE

3. You convince your parents that any amount over the five dollars you will pay for from the allowance you have saved. You were so excited when they agreed, that you called your friend immediately! It was Thursday, 4:45 pm. You needed to spend at least 45 minutes on your homework, but you talked long distance for 1 hour and 51 minutes! How much will you have to pay? Show all work and explain your solution in writing.

LONG MOWER

FOCUS: Application Activity
- Rate

PURPOSE: The student will . . .
- Devise and execute a plan to estimate for the data required, and
- Solve a multistep rate problem using the data.

STUDENT BACKGROUND: Students should be familiar with rates.

MATERIALS: LONG MOWER worksheet and calculator.

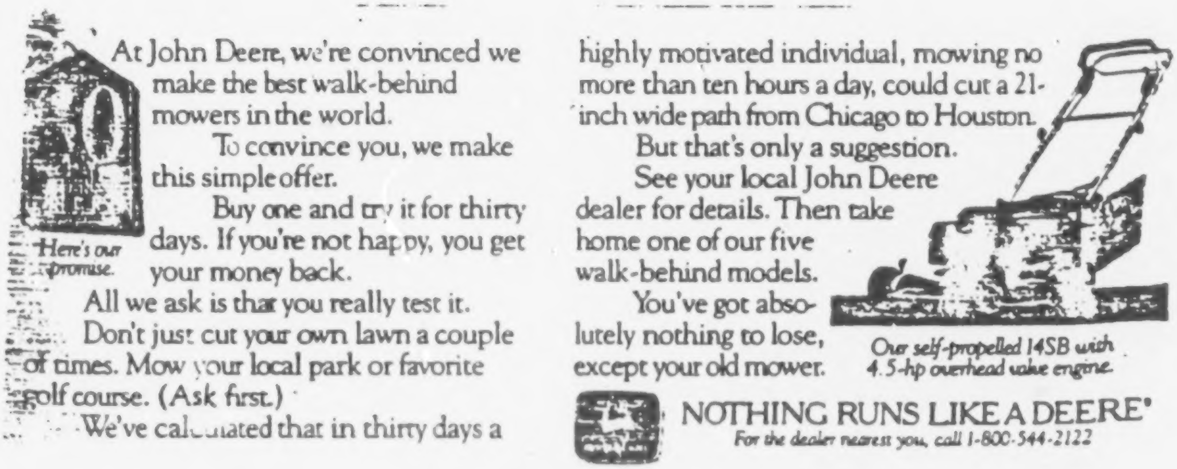
LESSON DEVELOPMENT: Students are to use the information in the advertisement and an estimate of the length of the 21-inch swath of lawn they would mow in some unit of time to calculate the distance from Chicago to Houston. They are then to check their results with some other source to determine their accuracy. Percent error could be introduced if it has not yet been, and students could find whose percent error is least. Absolute error comparisons could be made instead.

ANSWERS: Vary
Brisk walking speed [without a lawn mower] for a man is 6 km/hr[1] ≈ 3.73 mph. This would be 37.3 miles per day or 1119 miles in 30 ten-hour days at this pace. According to one atlas[2], the distance is 1067 miles. Students might discuss the accuracy of the information in the advertisement, whether it was intended to be taken literally--as a pace one could maintain for 30 ten-hour days while pushing a lawn mower, and whether they should write to John Deere to question their truth in advertising.

References:
[1] Diagram Group. (1980). *Comparisons.* New York: St. Martin's Press.
[2] Rand McNally. (1988). *Road atlas and travel guide.*

LONG MOWER

Use the information in this advertisement to calculate the distance from Chicago to Houston. After you report your results to the teacher or class recorder, find the actual distance in a travel atlas. Compare your results with those of your classmates.



At John Deere, we're convinced we make the best walk-behind mowers in the world.

To convince you, we make this simple offer.

Buy one and try it for thirty days. If you're not happy, you get your money back.

All we ask is that you really test it.

Don't just cut your own lawn a couple of times. Mow your local park or favorite golf course. (Ask first.)

We've calculated that in thirty days a highly motivated individual, mowing no more than ten hours a day, could cut a 21-inch wide path from Chicago to Houston.

But that's only a suggestion.

See your local John Deere dealer for details. Then take home one of our five walk-behind models.

You've got absolutely nothing to lose, except your old mower.

Here's our promise.

Our self-propelled 14SB with 4.5-hp overhead valve engine.

NOTHING RUNS LIKE A DEERE®

For the dealer nearest you, call 1-800-544-2122

PLAN AHEAD

FOCUS: Application Activity
- Rate

PURPOSE: The student will . . .
- Solve multi-step rate problems;
- Use diagramming, visualizing, and problem-solving planning in multi-step problems;
- Recognize that a problem contains insufficient information for solution; and
- Write a multi-step rate problem.

STUDENT BACKGROUND: The students should be familiar with rates and should be able to find a percent of a number.

MATERIALS: PLAN AHEAD worksheet

LESSON DEVELOPMENT: Although most of the problems are appropriate for individual or group work, Problem 4 asks for a group plan. If the students lack experience in making diagrams, planning, and describing solutions for multi-step problems, the class might work the first problem as a group with students suggesting the solution plan and various ways to diagram the situation. A student might be asked to summarize the steps used, following the solution, as a model for explanations required in other problems.

These problems provide an opportunity to distinguish between estimates and exact answers, an aspect which should be considered in the class discussion of the solutions for each problem. For example, the data in Problem 2 are clearly estimates, and mental computation and estimation would be appropriate for each step in the solution. The answers in Problem 3 require interpretation of the remainders, whether to take nearly a whole box more than is needed or to be just a little short, since the allowances are just estimates. Problem 8 contains insufficient data for solution. Students could suggest reasonable data and solve the problem, after the need for more information is discussed.

ANSWERS:

1. Cost - Earnings = Extra amount needed
   $84.79 - $75.80 = $8.99
   Extra needed ÷ hourly rate = # Extra Hours need to work
   $8.99 ÷ $3.79 = 2.37 hours, so would need to work 3 extra hours

2. 400 gallons needed for trip, so allow at least $440 for gasoline in addition to what individuals may estimate they would use while at the destination.
3. Just over 6 boxes, so students could decide if they would take a little in a smaller container, just forget the missing 3/4 cup or take an entire seventh box with plenty to spare. Estimating 1 cup means they would need a little more than 8 boxes, an extra 4 lbs.
4. Answers vary.
5. At $7.05 per record, $7.47 with tax, can afford 4 records.
6. Answers vary.
7. 0.825 hours or 49.5 minutes
8. Insufficient information (Need to know cost of movie ticket)

PLAN AHEAD

1. After you read the following problem, make a diagram to show your thinking in planning a solution. Show your work for solving the problem.

You know that you are going to a big party in a month, and you need something special to wear. While shopping, you have found the perfect outfit, priced $79.99. If you have a job at which you earn $3.79 per hour and normally work five hours each week, will you earn enough in four weeks to buy the outfit? Tax on clothing is 6%.

If so, how much will you have left for extras?
If not, how many extra hours would you need to work during the month?

2. In planning a cross-country auto trip, one of the major expenses to consider is gasoline. If your three-day trip is about 2400 miles one way, your van gets 12 mpg on the highway, and gas costs about $1.10 per gallon at your gas station, how much money should you allow for the gasoline for the entire trip? In some states, gasoline costs a little more than at home, and in some, a little less. You may add some mileage for driving you may do while at your destination. Show your work.

3. Part of your food assignment for your group's 5-day camping trip is to bring enough biscuit mix for everyone's biscuits, pancakes, and muffins for the entire trip. After discussing the menus and recipes you will use while camping, you've estimated that allowing $\frac{3}{4}$ cup of biscuit mix per day for each person is reasonable. Since 11 group members and two faculty advisors are making the 75-mile trip, how much biscuit mix should you take? One box holds 2 lbs. which is about 8 cups.

Suppose that, even though you had estimated that you needed only $\frac{3}{4}$ cup of biscuit mix per day for each person, you decided to allow 1 cup per person per day--just to make the math easier. Estimate the amount of extra biscuit mix you would have taken.

4. Your group has $130 for the day's fun at the ocean.
Jet ski rental rates are
$30 per hour for a 440 jet ski;
$40 per hour for a 640 jet ski;
$50 per hour for a Wave Runner; two people can ride it.
The minimum age is 14.
Roller skates or blades can be rented for $5 per hour for each pair.
Explain clearly how your group will spend the money you have. Briefly describe the way you reached your decisions.

5. **Before** you solve this problem, read it, visualize the steps for solving it, and briefly describe your plan for the solution (in writing).

You are planning to buy some records, but are not sure how many you can afford. Your friend has just bought two records of the same kind you want and a CD. He remembers that the cost was $25.98 before the clerk added the tax and that the CD was on special for $11.88. How many records will you be able to buy at the price he paid if you have $30.00? Be sure that you will you have enough money for the 6% sales tax.

6. Write a problem using a situation of your choice that requires planning and thinking similar to that you did in these problems. Make it reasonable for your classmates to solve, but be sure the solution requires more than one step.

7. A wrestler can't afford to gain any weight before the next meet. He wants to eat a $1\frac{1}{2}$ cup serving of ice cream today. The ice cream has 165 Calories per half cup. He knows he can burn about 600 Calories per hour running. How long will he have to run to burn off the calories from the ice cream before the weigh-in for his match? Before you start the calculations, estimate whether he will have to run for more or less than an hour.

8. You and three friends plan to go out for a snack after a movie; you need to plan ahead to be sure you can afford the food you want to order. A burger costs $2.59, onion rings are $0.99, and a shake is $1.69. If you started the evening with $9.65, do you have enough money to order a burger, onion rings, and a shake?

WANT AD

FOCUS: Application Activity
- Rate

PURPOSE: The students will...........
- Understand the meaning of rate, and specifically the translation of "per";
- Experience writing an ad and calculate the words per line and the number of lines in the ad;
- Calculate the cost of runnning the ad for a day and for a week; and
- Realize that multiplication can be used in calculating rate problems.

STUDENT BACKGROUND: Students should be able to multiply and divide decimals if calculators are not available.

MATERIALS: WANT AD worksheet

LESSON DEVELOPMENT:

$1.95 per line per day. 30 characters per line (includes spaces).

"Your family is moving to a new home. Mom wants you to sell your old bike, stereo, and ski equipment (or any other three items). All proceeds go directly into your pocket."

- Ask students to be creative and convincing, and to come up with all the details.
- Discuss these rates: cost per line, cost per day, and characters per line.
- In the written explanations, encourage students to use the words rate and per.

Closure: Discuss other real life situations where rate appears (cost per pound, miles per gallon, beats per minute).

ANSWERS:

1. Answers will vary. 2. Answers will vary. 3. Number of lines ÷30 4. Number of lines x $1.95 = cost per day. 5. Cost per day x 7 = cost per week. Extension: 75 + as many as 24 (spaces); 99÷30= 3.3. Approximately 4 lines. 4 x $1.95 = $7.80.

WANT AD

Newspaper rate for placing an ad:

$1.95 per line per day. 30 characters per line (includes spaces).

- Rewrite the rate for placing an ad substituting "for 1" in place of "per".
- Write an ad for the items you want to sell. Be sure to include the following information:
  Item
  Age of item/condition/description/make
  Cost
  Phone number

Write the ad exactly as it will appear in the newspaper (30 characters per line). Do not over-abbreviate and do not hyphenate words.

— —
— —
— —

Complete the following:

1. Count the number of characters (include spaces)______________
2. Calculate the number of lines in the ad.____________________
3. Write an equation showing how you can calculate the approximate number of lines knowing the number of characters ______________
4. Calculate the cost of your ad per day ______________________
Write an equation for the cost.___________________________
Explain the equation in words and use the appropriate language.
____
____
5. Calculate the cost to run your ad per week.________________
Write an equation for the cost ___________________________
Explain the equation in words and use the appropriate language.
____
____

EXTENSIONS: Susan writes an ad 25 words (93 characters) long. Find the total number (include spaces) of characters and calculate the approximate number of lines in the ad. What is the cost to run the ad for four days?

LET'S COMPARE

FOCUS: Meaning-Centered Lesson for Multiplication

- Comparison

PURPOSE: The students will . . .

- Compare lengths multiplicatively, comparing both the larger to the smaller and the smaller to the larger;
- Learn the equivalence of $a = bc$ and $c = \frac{1}{b}a$; and
- Express this relationship in writing.

STUDENT BACKGROUND: The students will need to know the meaning of a fraction and that multiplication by a fraction (less than 1) can be used to find part of a whole.

TEACHER BACKGROUND: Comparison is one of the primary uses of multiplication in the middle school. It is the basis for work with scale factors, units conversion, and proportional reasoning in addition to the basic comparison situation. It should be emphasized that the comparison can be made in either direction. The metric-English conversions are included to illustrate conversion between units, not to discourage "thinking metric."

MATERIALS: LET'S COMPARE worksheet, ruler, paper, and scissors for making paper strips.

LESSON DEVELOPMENT: Students make concrete comparisons to determine the multiplicative relationships between the lengths of strips of paper and express these relationships in equations. They will summarize the relationships they find.

EXTENSIONS: The relationship between mixed numbers and their reciprocals can be further investigated so that students can generalize their relationship. This lesson requires students to **generalize** only about integers and their reciprocals, although other numbers are used in the problems.

ANSWERS:

1. 5, 5, 5
   1/5, 1/5, 1/5
2. 3 1/2, 3 1/2, 3 1/2
   2/7, 2/7, 2/7
3. 2 1/2, 2 1/2
   2/5, 2/5,2/5,2/5, 2/5
4. 1/1000 inch thick; 1000 strands; 4
5. 1 2/3,1 2/3
   3/5, 3/5, 3/5, 3/5
6. About 3; about 1/3
7. 1 / 2
   For example, amount A is 50 times as much as amount B, so amount B is 1/50 as much as amount A.
8. 2/3 as big
9. Answers vary.

SOURCE: Adapted from

Mathematics Resource Project. (1977). *Number sense and arithmetic skills*. Palo Alto, CA: Creative Publications.

LET'S COMPARE

1. Cut strips of paper of equal widths as follows:
    One strip 1 inch long--label it Strip A;
    One strip 5 inches long-- label it Strip B.

Compare the lengths of the strips. How many times will Strip A fit side by side on Strip B? ___

Therefore, Strip B is ___ **times as long as** Strip A.

In an equation: $\text{Length}_B = \_\_\_ \times \text{Length}_A$.

Now compare the two strips another way:

The length of Strip A is what fractional part of the length of Strip B? ____

Therefore, Strip A is ____ as long as Strip B.

OR

The length of Strip A is ___ **of** the length of Strip B.

OR

In an equation: $\text{Length}_A = \_\_\_ \times \text{Length}_B$.

2. Cut strips of paper of equal widths as follows:
    One strip 2 inches long--label it Strip C;
    One strip 7 inches long--label it Strip D.

Compare the lengths of the strips. Mark off the length of Strip C on Strip D as often as you can. How many times will Strip C fit side by side on Strip D (including any fractional parts)? ____

Therefore, Strip D is ___ **times as long as** Strip C.

In an equation: $\text{Length}_D = \_\_\_ \times \text{Length}_C$.

The length of Strip C is what fractional part of the length of Strip D? ____

Therefore, Strip C is ____ as long as Strip D.

OR

The length of Strip C is ___ **of** the length of Strip D.

OR

In an equation: $\text{Length}_C = \_\_\_ \times \text{Length}_D$.

Explain the difference between the two ways you compared these two strips.

3. Use your ruler to compare an inch and a centimeter.

An inch is about ___ **times as long as** a centimeter.
In an equation: 1 inch ≈ ___ x 1 centimeter.

One centimeter is about what fractional part of one inch? ____
Therefore, a centimeter is about ____ as long as an inch.
OR
A centimeter is about ___ **of** an inch.
OR
In an equation: 1 centimeter ≈ ___ **x** 1 inch.

4. A human hair is about $\frac{1}{250}$ inch thick. Wool from the merino sheep is the most expensive wool available. It is $\frac{1}{4}$ as thick as a human hair. Is this thicker or thinner than human hair? How thick is it? ___ in.
How many strands of merino sheep wool, side by side, are needed to make one inch? _____
A human hair is about how many times as thick as a strand of merino sheep wool? _______

5. Use the following approximate scale to compare 1 kilometer and 1 mile.

|----|----|----|

1 Kilometer

|----|----|----|----|----|

1 Mile

One mile is about ___ **times as long as** 1 kilometer.
In an equation: 1 mile ≈ ___ x 1 kilometer.

One kilometer is about what fractional part of one mile? ____
Therefore, 1 kilometer is about ____ as long as 1 mile.
OR
One kilometer is about ___ **of** one mile.
OR
In an equation: 1 kilometer ≈ ___ **x** 1 mile.

6. The cheetah can sprint at 75 mph, although it soon tires at this speed. The fastest human at the 1988 Olympic games ran 27 mph. About how many times as fast as a human can a cheetah sprint? A human can run ___ as fast as a cheetah.

7. In comparison by multiplication, if one amount is twice as big as the second, how does the second compare with the first? ______

Give a general statement about the relationship of the second amount to the first when the first is any whole number of times as much as the second

.

8. If one amount is $1\frac{1}{2}$ times as big as the second, how does the second compare with the first? ____
Explain how you got your answer.

9. Write problems for a classmate to compare (in both directions).
Suggestions:
a) Use information about the numbers of recordings sold of the latest releases for your two favorite musical groups. Estimate numbers that would be appropriate if you do not know the exact numbers.
or
b) Use the amounts of time you spend sleeping and watching TV or studying, etc.
or
c) Use comparisons of your choice.

VERNAL FALLS TRAIL

FOCUS: Meaning-Centered Lesson for Multiplication

- Comparison

PURPOSE: The student will...

- Estimate twice as far, $\frac{2}{3}$ as much, and $\frac{3}{4}$ as far on a picture;
- Learn multiplication is the operation used in calculating multiplicative comparison word problems (with both numbers greater than one and less than one);
- Deal with extraneous information; and
- Generalize and express in written form what is learned.

STUDENT BACKGROUND: Students should be able to multiply fractions and should possess fraction number sense. Students should know the part-of-amount meaning for multiplication of fractions and associated vocabulary.

TEACHER BACKGROUND: Research shows that many students believe multiplication always makes bigger. In this lesson students experience that multiplication may make bigger or smaller depending on whether the multiplier is greater than one or less than one.

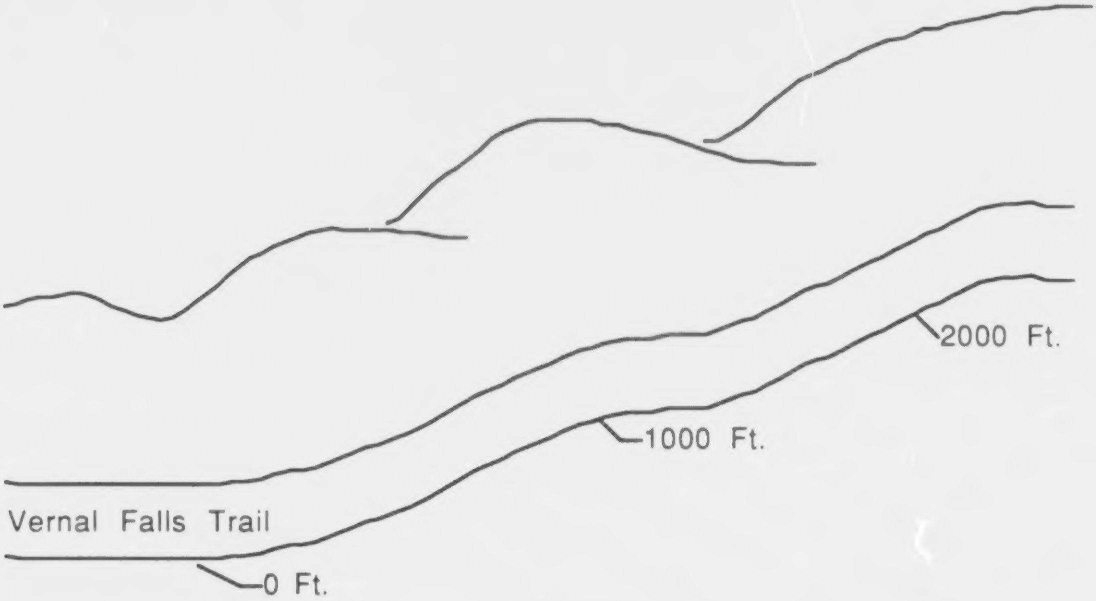
MATERIALS: VERNAL FALLS worksheet

LESSON DEVELOPMENT: Review with students which numbers are less than one and greater than one (if necessary). When approaching the problem, follow the order of the teenagers as given. Students should recognize that multiplication yields the correct answer for problem #3. Encourage students to continue to use the same operation in each instance. Ask students whether their answers are reasonable. Have students experiment with division (if it comes up) and check for reasonableness. Discuss with students multiplicative comparison word problems, perhaps asking them to come up with other examples. Make sure the written sentences specifically include a description of this type of word problem and the operation used every time.

ANSWERS: 1. Answers will vary. 2. Ahead. 3. 2400 ft. 4. 2 x 1200 5. Behind. 6. 1600 ft. 7. $\frac{2}{3}$ x 2400. 8. Behind 9. 900 ft. 10. $\frac{3}{4}$ x 1200. 11. Answers will vary. 12. Answers will vary. Extension: 2400 ft.

VERNAL FALLS TRIAL

On a warm sunny day in August, four teenagers, Jerome, Susan, Thomas, and Tran went hiking on the Vernal Falls Trail (a very steep climb in elevation) in Yosemite National Park. After 15 minutes, Thomas had hiked 1200 ft. Jerome had hiked twice as far as Thomas. Susan had hiked $\frac{2}{3}$ as far as Jerome, and Tran had hiked $\frac{3}{4}$ as far as Thomas. How far did Jerome, Susan, and Tran hike?



1. Estimate on the picture where each of the teenagers is after 15 minutes. Show Thomas and Jerome first.
2. Is Jerome ahead of or behind Thomas?________________________
3. How far did Jerome hike? ________________________________
4. Besides addition which operation works for this problem? _____ ____
5. Is Susan ahead of or behind Jerome?_________________________
6. How far did Susan hike?____________________________________
7. Using the same operation as in problem #4, write a mathematical expression that gives you the answer. __________________________
8. Is Tran ahead of or behind Thomas?___________________________
9. How far did Tran hike? ______________________________________

10. Write a mathematical expression that gives you the correct answer.____

11. Check your estimates. How close were you? Change your estimates if necessary.

12. In each case the distance hiked by one teenager was compared to the distance hiked by another teenager. No matter what the factor, 2 (a number greater than one), $\frac{2}{3}$, or $\frac{3}{4}$ (numbers less than one), the same operation was used to find the answer. Write a sentence and generalize about the type of problem and the operation used to find the answer each time.

____

____

____

____

____

EXTENSION: A fifth teenager, Juan, joined the group a little late. After 15 minutes, he climbed 1 $\frac{1}{2}$ times as far as Susan did. How far did Juan hike?

CHILI COOKOFF TEACHER'S GUIDE

FOCUS: Application Activity for Multiplication and Division
- How-many-equal-amounts
- Sharing equally
- Scale factors

PURPOSE: The student will . . .
- Confront common **misconceptions** regarding the meaning and effect of division with numbers less than 1.
- Review other concepts related to meanings of operations as listed in the focus above.

STUDENT BACKGROUND: This lesson requires students to deal with extraneous information, deal in varied ways with remainders in whole number division, use fraction sense and meaning of division by a fraction in drawings, use a calculator to change a fraction to a decimal, and use scale factors in recipe adjustment problems. It is assumed that the students have previously encountered the idea that multiplication can make smaller when the multiplier is less than 1.

TEACHER BACKGROUND: Students often erroneously believe that multiplication always makes bigger and that dividing always produces a quotient smaller than the dividend. This lesson attempts to present division by a number less than 1 in a context students may readily identify as a division situation in spite of the usual misconception. When the result of the division less than 1 here is highlighted, this situation could be referred to in future, less obvious, division situations in which the misconception would be more likely to emerge.

MATERIALS: CHILI COOKOFF worksheet, cookbooks for reference, and provisions for actual Chili Cookoff, if desired.

LESSON DEVELOPMENT: The lesson may require **two or more** days.

For most classes this would be appropriately used as a small group activity rather than an individual one; the lesson involves considerable reading as well as writing of explanations and original problems, includes extraneous information, and requires decisions regarding remainders in division. These complications are in addition to the issue of the misconceptions mentioned above and the review of several operation meanings.

If students say multiply in Problem 1, Part f),since the answer is bigger, ask what is being done to the meat.
Additional work related to the misconceptions that may emerge related to Parts l) and m)--multiplication always makes bigger and division always makes smaller--is provided in the CHECKING HOMEWORK worksheet.

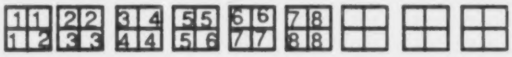
Following Part m), discuss the extraneous information in the problem. Students may suggest additional questions using this information.

In Problem 2, students need to deal with the remainder in a sensible way. In discussing Problem 3, establish the relationship between 1 1/2 and 3/4, arriving at the scale factor 1/2. Use the language used for this meaning of multiplication in class.

For Problem 7, students may use a cookbook and work in a group to write appropriate scale-factor word problems (7a) to be solved by classmates. If they have not recently discussed the equivalence of multiplying by 1/2 and dividing by 2, students may be puzzled by Part b.

Follow-up: The Checking Homework worksheet gives students opportunities to consolidate the information related to the effects of multiplying and dividing by numbers less than 1.

EXTENSIONS: Further recipes to increase amounts, perhaps for an International Food Fair on campus, with each booth selling the food representative of an ethnic group present on campus.

ANSWERS:
1a. 9 groups if each gets 1 lb.
1b. $9 \text{ lb} \div 1 \frac{\text{lb}}{\text{group}}$ --> 9 groups
1c. More than nine groups
1d. Each group gets less than a pound, so there will be more than 9 groups.
1e. 

etc., so 12 groups can be formed.
1f. Divide.
1g. The meat is being divided into equal amounts.
1h. $9 \div \frac{3}{4} = 9 \times \frac{4}{3} = 3 \times 4 = 12$ groups.
1i. 1, 8, 9, 12 are correct.

1j. $3 \div 4 = 0.75$; $9 \div 0.75 = 12$.

1k. 2, 3, 5, 7 are correct.

1l. Greater than the dividend because there are more than 9 three-fourths in 9.

1m. Less than the other factor because each three-fourth is less than one, so 9 three-fourths is less than 9.

2. Six students remain after 2 are put into each group, so 6 groups have three people.
3. Since $\frac{3}{4}$ is $\frac{1}{2}$ of $1\frac{1}{2}$, use half of the original amount of each ingredient.
4. We can make $\frac{3}{4}$ of a recipe. Three-fourths of 4 cans is 3 cans.
5. Scale factor is 2. Double each ingredient.
6. Multiply the ingredient amounts in the original recipe by the scale factor to find the amounts in the adjusted recipe. The scale factor is used to compare two amounts, the ingredients of the original recipe and the adjusted recipe, by multiplication.

Checking Homework is a worksheet with problems similar to those completed in the lesson. A fictitious student, Adam, made errors revealing some of the misconceptions anticipated in this lesson. Students are to discover the errors and explain what the student who made them was probably thinking and how the error could be explained to him so he would not make it again.

1. Adam answered correctly since the divisor was more than 1 and division would give the smaller answer he sought.

2. Adam should choose $15 \div \frac{2}{3}$ because the ribbon is to be **divided** into equal-sized lengths.

3. Adam should use $\frac{1}{3} \times 45$ or $45 \div 3$ to get the answer 15 ft. of paper for each group (Review this equivalence as needed). He correctly divided--$15 \div 1\frac{3}{4}$--to find the number of flowers each group can make. He should explain that each group can make 8 flowers and have a little paper left over since $8\frac{4}{7}$ flowers doesn't make sense.

4. Multiplying 3 by a number smaller than one gives a product less than three. Adam should divide--$3 \div \frac{3}{4} = 3 \times \frac{4}{3} = 4$--or just use the drawings he made, as he did.

CHILI COOKOFF

A class of 30 students has decided to have a chili cookoff. The class will form groups. Each group will make enough chili for themselves and for the three judges to taste. They had just $17.07, enough for nine pounds of ground beef. Each group will chose or invent a chili recipe to use $\frac{3}{4}$ of a pound.

1. The first thing they will do is decide on the number of groups that can be formed, allowing $\frac{3}{4}$ of a pound of ground beef for each group.

a) If one pound were give to each group, how many groups could be formed? ____________

b) How did you decide? ________________________________

____

____

c) Since each group is to get $\frac{3}{4}$ of a pound of ground beef, will there be more than nine groups or fewer than nine groups? ____________

d) How did you know? ________________________________

____

____

e) Make a drawing to show the nine pounds of ground beef distributed in $\frac{3}{4}$ lb. amounts.

How many groups can be formed? ____________

f) What operation (+, -, x, ÷) would you use to find the number of groups? ____________

g) Why? ____

____

____

h) Find the number of groups using the operation you chose. Show your work.

Did you get the same result as in Part e)? __________

i) In the following, check the squares for the expressions that mean the same as $\frac{3}{4}$.

1) ☐ $3 \div 4$
2) ☐ four divided by three
3) ☐ 1.33
4) ☐ $4 \div 3$
5) ☐ $1\frac{1}{3}$
6) ☐ $\frac{4}{3}$
7) ☐ 3.4
8) ☐ three divided by four
9) ☐ $4\overline{)3}$
10) ☐ $3\overline{)4}$
11) ☐ 3 Remainder 4
12) ☐ 0.75

j) How would you find the answer using a calculator? __________
____
____

Did you get the same answer as in Part h)? __________

k) Check the squares for the ways you could use to find the number of groups in the chili-making contest.

1) ☐ $9 \times \frac{3}{4}$
2) ☐ nine divided by three-fourths
3) ☐ $9 \div \frac{3}{4}$
4) ☐ 0.75×9
5) ☐ $0.75\overline{)9}$
6) ☐ $\frac{3}{4} \div 9$
7) ☐ $9 \div 0.75$
8) ☐ nine times three-fourths
9) ☐ $9\overline{)0.75}$

l) Does dividing by $\frac{3}{4}$ give an answer greater than or less than the dividend (9 is the dividend in $9 \times \frac{3}{4}$)? ____________________
Why? ____
____
____

m) Does multiplying by $\frac{3}{4}$ give an answer greater than or less than the other factor (9 is the other factor in $\frac{3}{4} \times 9$)? ____________________
Why? ____
____
____

2. Now that you know how many groups you will have, how many of the 30 students will be in each group? ____________________________

____

Explain how you reached your answer.

____

____

____

3. If your group is using a chili recipe that calls for $1\frac{1}{2}$ pounds of ground beef, what amounts of each of the other ingredients will you need to use in adjusting the following recipe (since you have just $\frac{3}{4}$ pound of ground beef)?

Original Recipe	Adjusted Recipe	
$1\frac{1}{2}$ lbs. ground beef	$\frac{3}{4}$ lb.	Ground beef
24 oz. chili beans	________	Chili beans
One-half cup chopped onion	________	Chopped onion
One teaspoon salt	________	Salt
One-fourth teaspoon chili powder	________	Chili powder
$2\frac{1}{2}$ lbs. canned tomatoes	________	Tomatoes

How did you decide? ______________________________________

____

____

4. If your group is using a chili recipe that calls for 1 lb. ground beef and 4 small cans of chili beans, how many cans of chili beans will you use (since you have just $\frac{3}{4}$ lb. of ground beef)? ________________

____

____

5. Suppose you are at home cooking for a larger group. You have 4 pounds of ground beef, and the recipe calls for 2 pounds. If the recipe calls for one-half teaspoon of chili powder and three-fourths cup of onions, how much of each will you use with 4 pounds of meat?

____________ chili powder ______________ chopped onion

How did you decide? ______________________________________

____

____

6. Summarize the way to use a scale factor to determine the amounts in an adjusted recipe? ____________________________________

____

____

7. a) Using a recipe for your favorite food, write problems for your classmates to do for three of the following expressions:

$2 \times 1\frac{1}{2}$ $\qquad 6 \div \frac{1}{2}$ $\qquad 6 \times \frac{1}{2}$ $\qquad 6 \div 2$ $\qquad 2\frac{1}{2} \times 2$

Prepare an answer key to use in checking the work of the group who works your problems.

b) For which of the expressions in Part a) could you have written the same word problem? ________________________________
Why? ____

____

CHECKING HOMEWORK

Adam did these homework problems; you are to check his work and his answers. If you find any mistakes, explain what Adam was probably thinking and explain his errors so he won't make them again.

1) The class has $7\frac{1}{2}$ yards of fabric for making 3 tablecloths for the chili-making contest. How much fabric can be used for each cloth? Explain why you chose the operation you chose.

$$7\frac{1}{2} \div 3 = \frac{15}{2} \times \frac{1}{3} = \frac{5}{2} = 2\frac{1}{2} \text{ yds. each}$$

I divided because I am making 3 equal pieces of the same size and the answer should be smaller than $7\frac{1}{2}$.

2) They have 15 yards of ribbon to make bows for decorating the table. If each bow uses $\frac{2}{3}$ yd. of ribbon, how many bows can the class make? Check the square for the operation you would use to solve the problem and explain your choice.

☐ $15 + \frac{2}{3}$ ☐ $15 - \frac{2}{3}$ ☐ $15 \times \frac{2}{3}$ ☐ $15 \div \frac{2}{3}$

I would multiply because I am making equal pieces of the same size and the answer should be larger than 15 since each bow uses less than 1 yard.

3) Three groups of four each are making paper flower decorations for the room. Each group has $\frac{1}{3}$ of the 45-foot package of paper to use. If each flower requires $1\frac{3}{4}$ ft. of paper, how many flowers can each group make?

$45 \div \frac{1}{3} = 15$ feet of paper for each group.

$15 \div 1\frac{3}{4} = 15 \times \frac{4}{7} = 60 \div 7 = 8\frac{4}{7}$ flowers.

4) How many recipes of chocolate chip cookies can you make for your party if you have three 1-pound bags of chips and your recipe calls for 12 oz.? Explain why you did the problem as you did it.

12 oz. = $\frac{3}{4}$ lb.

$3 \times \frac{3}{4} = 4$ recipes

I would multiply because I am using equal amounts of the same size and the answer should be larger than 3 since each recipe uses less than 1 pound.

UNIVERSAL SPORTS

FOCUS: Application Activity for Multiplication and Division

- Comparison
- Missing factor

PURPOSE: The student will . . .

- Learn to use multiplicative comparison to compare weights on Earth with those on celestial bodies with gravity different from Earth's;
- Use the inverse relationship between the height one is able to jump on a planet and the gravity of the planet;
- Distinguish between multiplicative and additive comparisons; and
- Use missing factor division in multiplicative comparison situations.

STUDENT BACKGROUND: Students will need experience with the part-of-a-whole meaning for multiplication. Students need to be able to convert between feet and inches and feet and between pounds and ounces and pounds as well as to round numbers. They need to solve for c from equations of the form a = bc.

TEACHER BACKGROUND: The quantitative inverse relationship between weight on a planet and the distance one could jump with the same effort on the planet may be difficult.

MATERIALS: UNIVERSAL SPORTS worksheet, and calculators.

LESSON DEVELOPMENT: Students may work in groups with each student answering the questions for the student's own weight and jumping ability after the group discusses the method to use. Be sure students understand what each column in the table represents. They should come to discover that the decimal and fractional values in the column "Times as High... " are approximately equal. The teacher should carefully lead the discussion throughout the example using the moon weights and jumping heights compared to those on Earth. Additional information about the planets and the reasons for the differences in gravity may be provided, or this lesson could be coordinated with the science teacher's presentation of such information. Although students are not always asked to explain the reasons for their answers in writing, these reasons should be elicited during the class discussion of the problems.

EXTENSIONS: Explore the reciprocal relationship between the numbers in the columns for "Gravity compared to Earth's" and "Times as high as jump on Earth" in more depth. Can students find this relationship without going through the comparison of weights and inverting it to find the second value?

ANSWERS:

4. a) Moon weight = 0.16 x Earth weight
   b) Earth weight ÷ Moon weight = 6.25
   c) Answers vary
   d) Answers vary
   e) Earth jump height x 6.25 = Moon jump height
2. 48.70 feet
3. a) 0.88
   b) higher; less gravity
   c) Venus weight = 0.88 x Earth weight
   Earth weight ÷ Venus weight = 1.14
   d) Yes; Louise's Earth weight = 1.14 x her Venus weight, too.
   e) Weigh 1.14 times as much on Earth as on Venus, so can jump 1.14 times as high on Venus as on Earth.
   f) 7.60 feet (1.14 x 6.67 ft)
4. Weigh more on the sun; Sun weight = 27.9 x Earth weight
   0.04 x Earth jump height = Sun jump height; less than jump height on Earth. Reasonable because weigh so much more on the sun.
5. 10 ft. ÷ 2.63 = Earth jump height. Earth jump height less than on Mars;
   10 ft = 2.63 x Earth jump height.
6. Mars where the gravity is less.
   Earth high jump height x 1.14 = Venus high jump height
   Earth high jump height x 2.63 = Mars high jump height
   Mars high jump height - Venus high jump height = Difference
7. 43.92 ÷ 16.25 = 2.70 times as high on Mercury as on Earth; expect to weigh less on Mercury.
8. Saturn's gravitational pull is greater than Earth's.
   89.75 ÷ 165 = 1.15. Greater than 1 since gravity > Earth's.
9. Greater gravitational pull, greater weight, less height can jump...
10. Answers vary.

Gravity throughout the Solar System

Location	Gravity compared to Earth's	Times as high as jump on Earth		Jump height compared to 3-ft jump	Your Weight	Your best Jump height	1988 Olympic Pole Vault Record*
Sun	27.90	0.04	$\frac{1}{25}$	≈$1\frac{1}{2}$ inch			
Jupiter	2.34	0.43	$\frac{5}{12}$				
Neptune	1.18	0.85	$\frac{5}{6}$				
Uranus	1.17	0.85					
Saturn	1.15	0.87					
Earth	**1.00**	**1.00**	1	3 '			19 ' $4\frac{1}{4}$ "
Venus	0.88	1.14	$1\frac{1}{7}$				
Mars	0.38	2.63	$2\frac{3}{5}$				
Mercury	0.37	2.70					
Moon	0.16	6.25	$6\frac{1}{4}$	≈18 ' 9 "			

SOURCE: Adapted from Mathematics Resource Project.

References:
Diagram Group. (1980). *Comparisons.* New York: St. Martin's Press.

Johnson, O. (Ed.). (1990). *The 1990 information please almanac.* Boston: Houghton Mifflin.

UNIVERSAL SPORTS

Your weight will change as you move about the solar system competing in interplanetary track and field events. Furthermore, your high jump and/or pole vault records will be affected by the gravitational attraction of the planet or moon you are visiting. The table below gives the effect of gravity around the solar system compared to Earth's gravity. For example, if you could jump a three-foot hurdle on Earth, you could jump one that is much higher on Earth's moon--because the gravity of the moon is only 0.16 the gravity of Earth.

1. Your weight on the moon is only 0.16 of your Earth weight.
a) Find your weight on the moon. _______ How did you find it?

Gravity throughout the Solar System

Location	**Gravity** compared to Earth's	Times as high as jump on Earth		Jump height compared to 3-ft Earth jump	Your Weight	Your best Jump height	1988 Olympic Pole Vault Record*
Sun	27.90	0.04	$\frac{1}{25}$	~$1\frac{1}{2}$"			
Jupiter	2.34	0.43	$\frac{5}{12}$				
Neptune	1.18	0.85	$\frac{5}{6}$				
Uranus	1.17	0.85					
Saturn	----						
Earth	**1.00**	**1.00**	1	3 '			19 ' $4\frac{1}{4}$ "
Venus	0.88	1.14	$1\frac{1}{7}$				
Mars	0.38	2.63	$2\frac{3}{5}$				
Mercury		----					
Moon	0.16	6.25	$6\frac{1}{4}$	~18 ' 9 "			

* Sergei Bubka, U.S.S.R.

Since you weigh less on the moon but have the same muscle development and make the same effort, you can jump much higher on the moon than on Earth.

b) Your Earth weight is about how many times your moon weight? __________ On the moon, you can jump approximately that many times as high as you can jump on Earth.

c) How high can you high jump on Earth? ________

(See the Olympic Records table below for the 1988 Olympic records for men's and women's running high jump to help you estimate, if you don't have any idea.)

d) How high will you be able to jump on the moon? _________

e) Show the equation you used.

To summarize:

The moon's gravity is 0.16 the gravity of Earth, so your weight on the moon is 0.16 your weight on Earth. Write the equation for this relationship:

AND

Your weight on Earth is about 6.25 times your weight on the moon, so you can jump about 6.25 times as high on the moon as you can on Earth. Write the equation that gives this relationship:

1988 Olympic Records

Event	Record	Winner
Men		
Running High Jump	7' $9\frac{1}{2}$"	Guennadi Avdeenko, USSR
Long Jump	28' $7\frac{1}{4}$"	Carl Lewis, U. S.
Pole Vault	19' $4\frac{1}{4}$"	Sergei Bubka, USSR
Weightlifting* (1977 record for Clean & jerk)	$564\frac{1}{4}$ lbs.	Vasili Alexeev, USSR
Women		
Running High Jump	6' 8"	Louise Ritter, U. S.
Long Jump	24' $3\frac{1}{2}$"	Jackie Joyner-Kersee, U. S.
Weightlifting* (c.1911 record for overhead lift)	286 lbs.	Katie Sandwina

*1988 Olympic Weights lifted not in Almanac

2. About how high should Guennadi Avdeenko be able to jump on the moon? ____

3. Find out how high Louise Ritter should be able to jump on Venus.
a) What part of their Earth weight do things weigh on Venus? ____
b) Compared to the height Louise can jump on Earth, should she be able to jump higher or not as high on Venus? ________
Why? ____
____

c) Compare your Venus weight and your Earth weight. How many times as great as your Venus weight is your Earth weight? _____

d) Even though you don't know Louise's weight on Earth or on Venus, can you find out how many times as great as her Venus weight is her Earth weight? ______

e) This is the comparison factor for comparing the height Louise can jump on Earth with the height she can jump on Venus. Explain why this is true. ____

____

____

f) Louise could high jump _____ ft. on Venus.

4. The numbers in the **Gravity table** in the column "Times as high as Jump on Earth" will tell you the comparison factor for comparing jumps in some other parts of the solar system with jumps on Earth.

On the sun, would you weigh more than your Earth weight or less? ___ How much would you weigh on the sun? ______ Show the equation you would use.

How high would you be able to jump on the sun? ____ Is this more or less than the height of your best jump on Earth? _____ Is this reasonable? ____ Why? ____________________________________

____

5. If a Martian could jump 10 feet high on Mars, how high could she jump on Earth? ____ Do you expect your answer to be more than 10 feet or less? ______ Show how you got the height of her jump on Earth.

6. On which planet, Mars or Venus, would you expect your high jump to be higher? ____ Why? ______________________________________

How much higher? ____

Explain how you found the difference in heights. ________________

____

____

7. If your friend, Bill, whose best pole vault on Earth has been 16 feet 3 inches, then vaults 43 feet 11 inches on Mercury, how many times as high can you expect to jump on Mercury as you did on Earth? ____ Would you expect to weigh more or less on Mercury than on Earth? ____

8. If Bill's weight on Earth is 165 pounds and his weight on Saturn is 189 pounds 12 ounces, how does Saturn's gravitational pull compare with Earth's? _______ Put the comparison factor in the **Gravity table** in the column "Gravity compared to Earth's." Should the comparison factor be greater than 1 or less than 1? _______

9. Summarize the relationships you have found among gravitational attraction, weight, and height you can jump on any planet in the solar system.

10. Using information you have not used so far from the Solar System table or from the Olympic Records table, write three word problems for your classmates to solve. Prepare an answer key to use in checking their answers to your problems.

NEW BEDROOM

FOCUS: Meaning-Centered Lesson and Application Activity
- Scale

PURPOSE: The students will...
- Learn that there is a multiplicative change when dealing with enlargements; and
- Gain an appreciation for how mathematics is used in real-life situations.

STUDENT BACKGROUND: Students should be familiar with conversions among inches, feet, and yards.

MATERIALS: NEW BEDROOM worksheet, ruler, yardstick, scissors, glue or tape.

LESSON DEVELOPMENT: The worksheet may be assigned as individual or group work. In either case there should be a class discussion on the meaning of scale. Encourage students to use appropriate language such as scale, scale factor, blueprint, etc. in their written work. Discuss other situations where scales are used, (maps, photo enlargements). Review with students that 3.75 feet does not mean 3 feet and 75 inches. Discuss how to convert such mixed decimals into the correct measure.

If the extension problem is used, students may present their blueprints to the class and display them on the bulletin board.

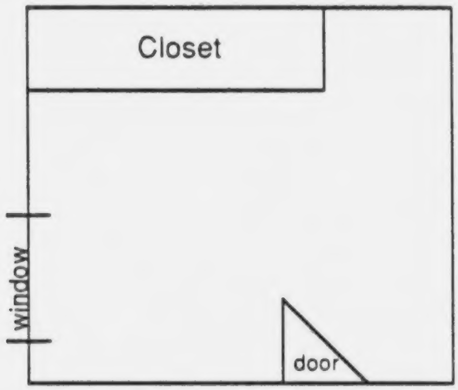
ANSWERS:
1. 12, 3, 36
2. $2\frac{1}{2}$", $\frac{1}{2}$"
3. A. $2\frac{1}{2}$", B. 150", $152\frac{1}{2}$", $147\frac{1}{2}$", 375 sq.", 60
   C. $\frac{1}{2}$", D. 30", $30\frac{1}{2}$", $29\frac{1}{2}$", 15 sq.", 60
4. The life-size measurement divided by the blueprint measurement is always 60.
5. 1/2 inch equals 60 inches (or 5 feet) or any other equivalent scale: 1 inch equals120 inches (or 10 feet). Discuss which one of these scales seems more reasonable to use for this blueprint.

6. Multiply the blueprint measurement by 60
7. Feet or yards
8. 3 feet 9 inches, (or 45 inches)
9. $12\frac{1}{2}$ feet by $11\frac{1}{4}$ feet, (or 150" by 135", or 12' 6" by 11' 3")
10. Division, 60
11. No, life-size distance between closet and window is $3\frac{3}{4}$ feet.
12. Larger
13. (Answers will vary. See blueprint on worksheet with furniture glued on it).
14. Answers will vary

SOURCE: Adapted from Math Lab Junior High, Action Math Associates, Inc.

NEW BEDROOM

Your parents have decided to make a room addition to your house and it is going to be YOUR NEW BEDROOM ! The architect left the blueprints for you and your family to look over, but there was one thing missing. The scale was not included. The only measurements you have are for the 12 $\frac{1}{2}$ foot long wall on the house where the room will be added, and the 2 $\frac{1}{2}$ foot wide door entering the bedroom. Can you figure out the scale?



1. Review
   Fill in this chart:
   ________ inches = 1 foot
   ________ feet = 1 yard
   ________ inches = 1 yard

2. On the blueprint, measure the length of the bedroom wall that will connect to the rest of the house. ______ Measure the width of the door. ______

3. Fill in this chart:

	Blueprint	Life-Size (in inches)				
Wall Common to Room and House	A	B	A + B	B - A	A x B	B ÷ A
Width of door	C	D	C + D	D - C	C x D	D ÷ C

4. From the chart above, what is the common relationship you see between the blueprint measurements and the life-size measurements? Explain in words.

____

____

____

5. What is the scale? ______ represents ______

6. How would you figure out the life-size measurement from any measurement on this blueprint ?

____

____

7. Would it be more realistic to express your answers in inches or feet or yards if you were finding out the life-size length of a wall?

____

8. What will be the actual width of the window in your bedroom? Show work below. Does your answer seem reasonable?

____________________ .

9. What will be the dimensions of your new bedroom?

____

10. How would you figure out a measurement on this blueprint from any life-size measurement? ____________________

11. Suppose you currently have a desk that measures 4 feet by 2 feet 11 inches. Will it fit between the closet and the window? Show your work and explain your reasoning in writing.

____

____

____

12. Would your bedroom be larger or smaller if the scale was $\frac{1}{4}$ inch equals 60 inches, instead of $\frac{1}{2}$ inch equals 60 inches?

_______________ .

HOMEWORK:

13. Measure the length and width (not height) of the furniture you currently have in your bedroom at home. Draw the furniture to the scale of this blueprint. Cut the drawings out and glue them to the blueprint the way you would like to have your new bedroom arranged.

14. Does everything fit? If money were no object, what new furniture might you like to have in your room? Would it fit?

____

____

____

____

EXTENSION: Make a blueprint of any room in a house. Use a different scale from the one used on this worksheet. Include doors, windows, and furniture. Make it as detailed as you would like it to be.

CREATE A HOLLYWOOD MONSTER

FOCUS: Meaning-Centered/Application Activity for Multiplication

- Scale
- Area
- Volume
- Proportionality

PURPOSE: The students will . . .

- Discover the multiplicative relationship (vs additive) between corresponding sides of similar figures;
- Learn the meaning and application of scale factors;
- Review area and volume;
- Learn the relationship between areas of similar figures;
- Learn the relationship between volumes of similar figures;
- Apply these ideas with respect to a student-created monster; and
- Begin/continue to develop the concepts of proportionality.

STUDENT BACKGROUND: The student will use measurement skills and the concepts of area and volume. They should be able to find areas for standard geometric figures. They will need fraction and decimal number sense and estimation skills. Ability to convert standard measurements is assumed. Students need to know the following vocabulary: corresponding parts, congruent angles.

TEACHER BACKGROUND: Even students who can use ratio and proportion to solve problems often do so without appreciating the multiplicative (vs additive) relationships involved. This lesson attempts to help students discover that figures whose sides have a constant difference are not necessarily similar, while figures whose sides have a constant quotient (ratio) are similar.

The difference in effects of a scale factor on length, area, and volume is difficult for students to master. In an effort to make the relationships comprehensible, they are led to discover these effects for themselves, and to apply them in situations that relate to the monster they create in Part 1.

MATERIALS: Graph paper, CREATE A HOLLYWOOD MONSTER worksheet, centimeter rulers, wooden or plastic cubes (up to 64 per group, but different groups may need them at different times).

LESSON DEVELOPMENT: The parts of the lesson not related to the student's own monster could be done in groups with group reports to summarize the findings shared at each stage. The lesson might extend for three days, one for similarity, one for area of similar figures, and one for volume.

Students first create a monster to engage their interest in the topic of scale and, more generally, proportionality. It is important to be sure that **students** are able to **distinguish** between figures that are similar and those that are not, **before they complete the table** on similar figures and draw conclusions about the necessary relationship for similarity. Decide as a group if the second figure in each pair could be an enlargement of the first. Students should measure the figures in Pairs 3 - 8 in centimeters to complete the table. The question on regular polygons may be an extension question. Before students begin the kite and house enlargement problems, review the equivalence of $\frac{a}{b} = c$ and $a = cb$. Problem 6 (Monster Scale Factor section) may require some discussion as to what is necessary to answer the question. Perhaps through questioning they can determine that they must first find the scale factor relating King Kong and the 18 inch model used in the movie then apply that scale factor to the height of their own monster to determine the appropriate model size for their monster to use with the King Kong props.

EXTENSIONS: Use ratios of circumferences of their own wrist/thumb, neck/wrist, and waist/neck to predict sizes of their monster. (AIMS)

ANSWERS:

SIMILAR FIGURES?

Pair	**Similar?** YES or NO	s_1 (cm)	S_1 (cm)	s_2 (cm)	S_2 (cm)	$S_1 - s_1$	$S_2 - s_2$	$S_1 \div s_1$	$S_2 \div s_2$	Scale Factor **If Similar**
1	NO	2	7	3	8	5	5	3.5	2.67	XXX
2	YES	2	10	3	15	8	12	5	5	5
3	YES	2	6	3	9	4	6	3	3	3
4	NO	2	5	3	6	3	3	2.5	2	XXX
5	NO	1	5	3	7	4	4	5	2.33	XXX

6	YES	2	3	4	6	1	2	1.5	1.5	1.5

Matching sides of similar figures have the same ratio, not the same difference.

$S_1 \div s_1 = S_2 \div s_2$ in similar figures

Pair	Similar? YES or NO	s_1 (cm)	S_1 (cm)	s_2 (cm)	S_2 (cm)	$S_1 - s_1$	$S_2 - s_2$	$S_1 \div s_1$	$S_2 \div s_2$	Scale Factor If Similar
7	NO	2	5	2	5	3	3	2.5	2.5	XXX
8	YES	2.5	5	2.5	5	2.5	2.5	2	2	2

Constant scale factor for sides is not sufficient. The corresponding angles must be congruent also.

Students may draw figures similar to the pairs given, or triangles, etc. In the nonsimilar pair, the difference in the lengths of corresponding sides is constant, and in the similar pair, the ratio of lengths of corresponding sides is constant.

No. All regular polygons with the same number of sides are similar.

1.

Small kite	Large kite
2	5
2 4/5	7
4	10
4 2/5	11

2.

Small house	Large house
1	1.5
1.5	2.25
3	4.5
4	6
4	6
6	9

3. Scale factor for the two figures: 5/3

Small Figure	Large Figure
2	3 1/3
3	5
4	6 2/3
5	8 1/3
6	10
9	15

4. Drawing height = Scale factor x Full-size height

Scale factor = Drawing height ÷ Full-size height

or

Full-size height ÷ Scale-drawing height = scale factor

Everything will be $\frac{\text{Scale-drawing height}}{\text{Full-size height}}$ of its actual size.

5. Scale factor for props = 2 feet ÷ Full-size height
Multiply this scale factor times the normal height of any prop to determine the model size for the prop.

6. King Kong:
Scale factor = 30 feet ÷ 18 inches = 30 ft. ÷ 1.5 ft. = 60 ÷ 3 = 20
Varies; Use: Full-size height ÷ **Model height** = 20; find model height.

7. Appetite:
40-foot monster eats 8 per day (But see Volume for more discussion of these questions.
50-foot monster eats 10 per day
10-foot monster eats 2 per day
15-foot monster eats 3 per day

Areas of Similar Figures

Pair	s_1 (cm)	S_1 (cm)	Scale Factor $S_1 \div s_1$	area small figure	AREA Large Figure	AREA ÷ area	(Scale Factor)^2
2	2	10	5	6	150	25	25
6	2	3	1.5	12	27	2.25	2.25
9	2	5	2.5	4	25	6.25	6.25
10	1	3	3	π	9π	9	9

8. The ratio of the areas is the square of the scale factor.

9. a) Blanket for model related as square of scale factor relating drawing size and model size. Students should determine amount of fabric needed based on fabric width they want to use.
b) Blanket for life-size monster related as square of scale factor relating drawing size and life-size. Find amount of fabric needed as in Part a).

10. Monster life size ÷ average man size = scale factor
(Scale factor)^2 x 6 pounds = # pounds monster's skin weighs

VOLUME

Cube	S_1	S_1	Scale Factor $S_1 \div 1$	volume small figure	VOL Large Figure	VOLUME ÷ 1	(Scale Factor)^3
2 x 2 x 2	1	2	2	1	8	8	8
3 x 3 x 3	1	3	3	1	27	27	27
4 x 4 x 4	1	4	4	1	64	64	64
n x n x n	1	n	n	1	n^3	n^3	n^3

11. The ratio of the volumes is the same as the scale factor cubed.

12. (Monster's height ÷ Human's height)^3 = Monster's volume ÷ Human's volume
= Monster's weight ÷ Human's weight

13. Note: Billstein and Trudnoski suggest that students suppose that Godzilla's thigh bone could support weight up to ten times the weight of a twenty-foot animal to determine if his thigh bone could support his weight. They ask if Godzilla could be tall enough to look in windows of the upper stories of a ten-story building (possibly 100 feet). They provide other information relevant to the discussion of this problem.

> Galileo (1564-1642) predicted that the tallest tree could not exceed 300 feet. Giant sequoias have grown as tall as 360 feet. The bark is up to 2 inches thick, the trunk is 30 feet in diameter, and some trees are over 3000 years old.
> A fish, in doubling its length, multiplies its weight by about eight. A fish [approximately] doubles its weight in growing from four to five inches long [4^3 is 64, 5^3 is nearly twice that].

14. Determining appetite from length was not appropriate. We should cube the scale factor for height to determine relative weights, then base the appetite on this. In determining appetite, we would need to consider other factors, such as activity level, as well.

15. Answers vary.

SOURCE: Adapted from

Billstein, R. & Trudnoski, J. (1989). NCTM Student Math Notes: Godzilla®: Fact or fiction? In J. W. Lott (Ed.), *News Bulletin*, November, 1989.

Hart, K. M. (1984). *Ratio: Children's strategies and errors.* London: NFER-NELSON.

Wiebe, A. (1988). *Designing the giant's coat.* AIMS Education Foundation.

References:

Diagram Group. (1980). *Comparisons.* New York: St Martin's Press.

CREATE A HOLLYWOOD MONSTER

Someone must earn a living by creating monsters since there are so many in TV cartoons, video games, and movies. This is your chance to create a monster that will get the attention of Hollywood. On graph paper, draw the monster (or monstress) you think can make you rich. Name your monster, and describe any special characteristics it has. Provide any additional information that might help sell your idea--possible settings in which you see your monster, special abilities it has, its personality....

Name ____

Characteristics: ____

____

What is your monster's **full-size** height? __________

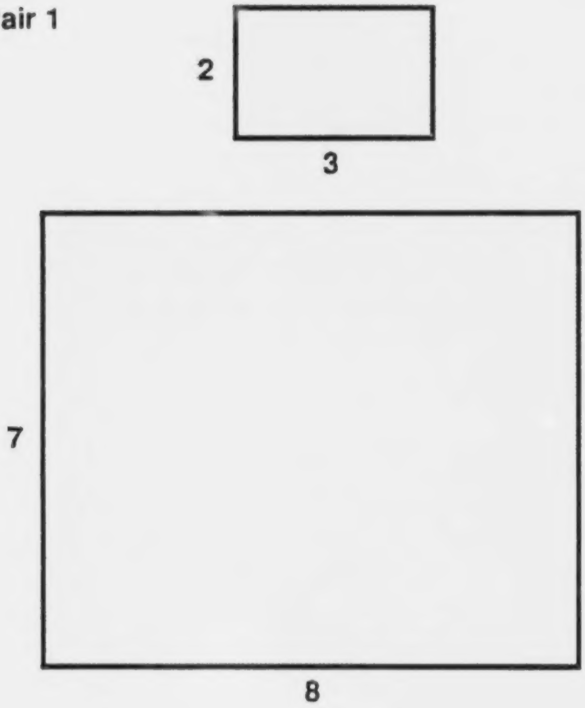
Similarity

Now let's explore some ideas in mathematics to decide if your monster could ever exist! **Similar** in mathematics has a more precise meaning than it has in normal conversation. Similar figures are figures with **exactly** the **same shape**. If two figures are **mathematically** similar, one could be an exact enlargement of the other. There are eight pairs of figures following the table. Decide if the figures in each of the pairs are similar. Measure the figures in Pairs 3 to 6 in **centimeters** to complete the table with the information about the lengths of the sides and the relationships between these lengths. The small letters refer to the smaller figure in each pair; the large letters refer to the matching parts (corresponding sides) in the larger figure.

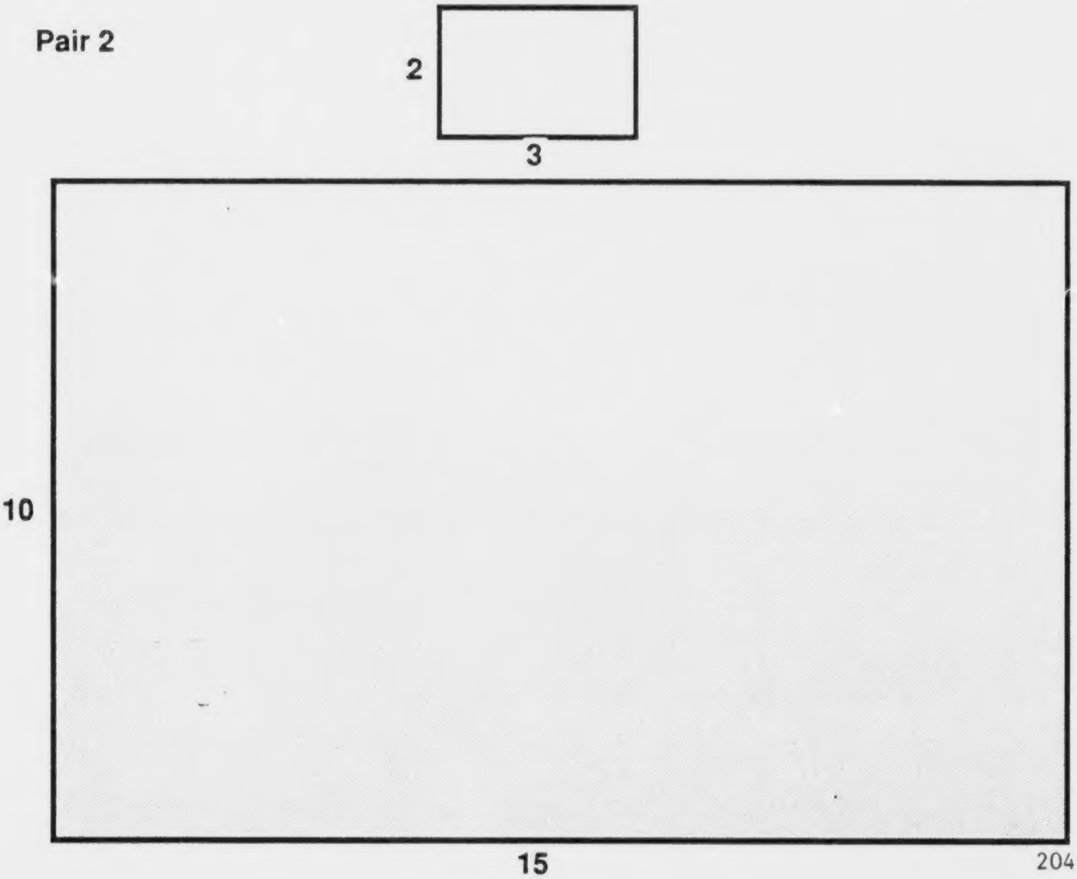
SIMILAR FIGURES?

Pair	**Similar?** YES or NO	s_1 (cm)	S_1 (cm)	s_2 (cm)	S_2 (cm)	S_1-s_1	S_2-s_2	S_1+s_1	S_2+s_2	Scale Factor **If Similar**
1	NO	2	7	3	8					XXX
2	YES	2	10	3	15					
3										
4										
5										
6										

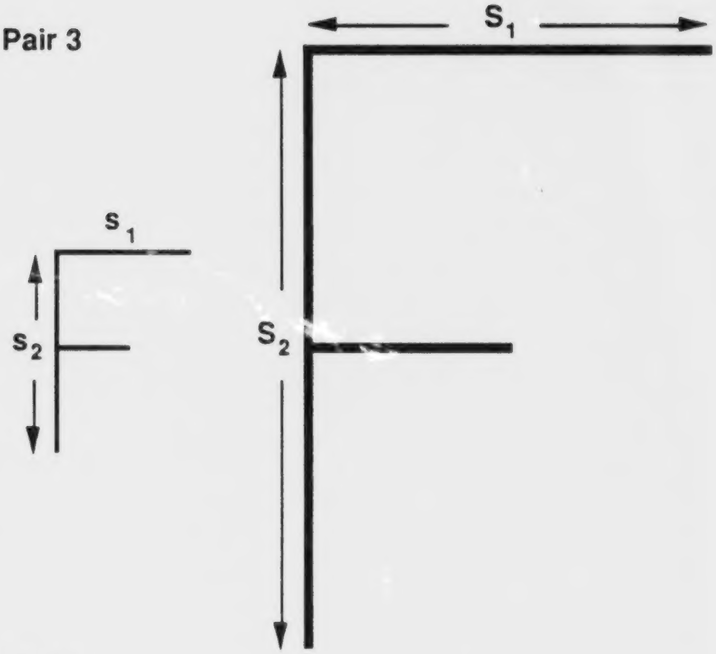
Pair 1



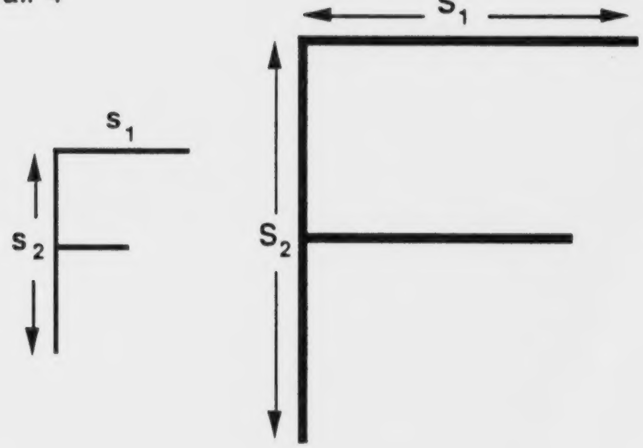
Pair 2



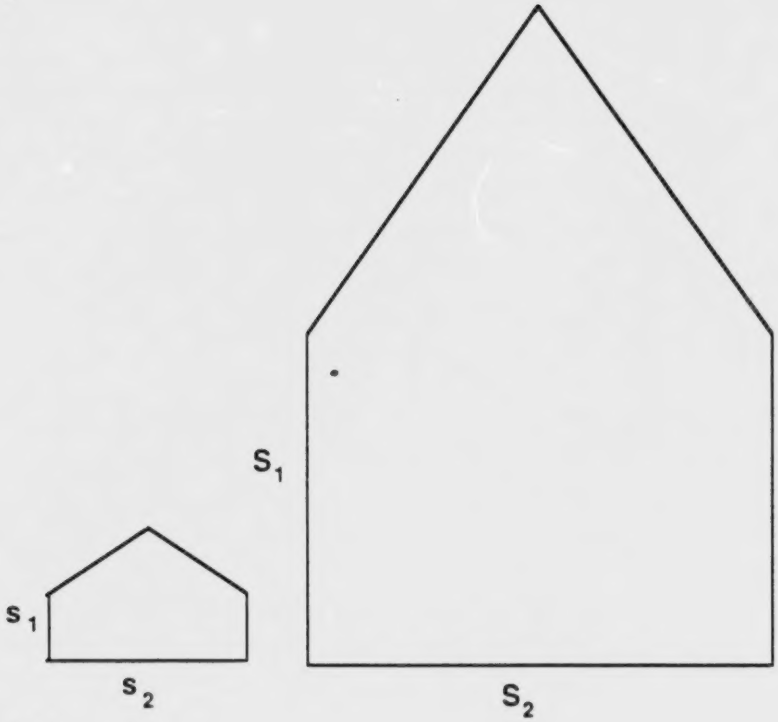
Pair 3



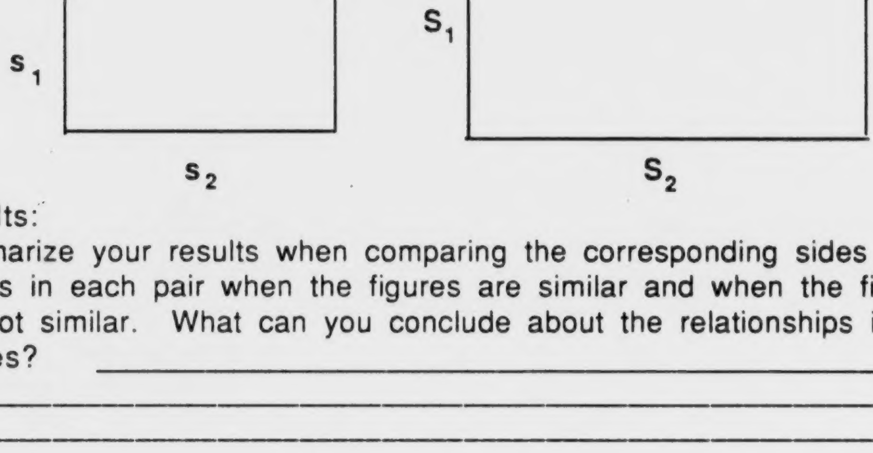
Pair 4



Pair 5



Pair 6



Results:

Summarize your results when comparing the corresponding sides of the figures in each pair when the figures are similar and when the figures are not similar. What can you conclude about the relationships in similar figures? ______________________________

____

____

Does $S_1 - s_1 = S_2 - s_2$ or $S_1 \div s_1 = S_2 \div s_2$ in similar figures?

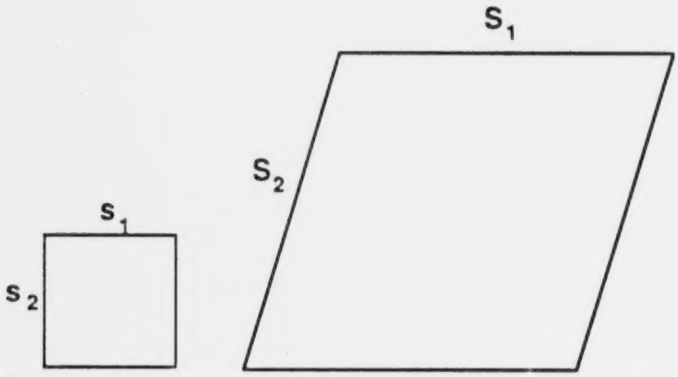
____

This constant quotient for the matching sides of two similar figures is called the **scale factor** for the two figures.

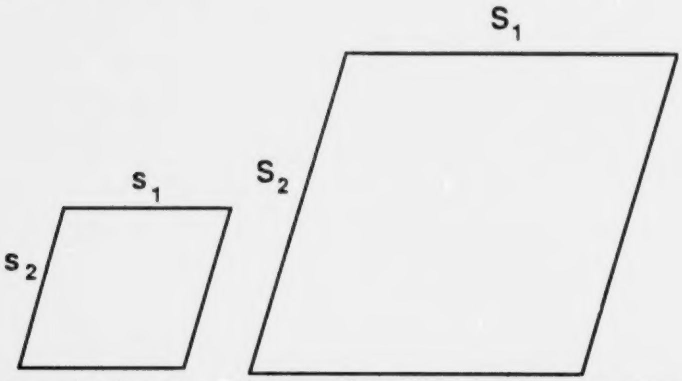
Now complete the table below for the figures in Pairs 7 and 8.

Pair	**Similar?** YES or NO	s_1	S_1	s_2	S_2	S_1-s_1	S_2-s_2	$S_1 \div s_1$	$S_2 \div s_2$	Scale Factor **If Similar**
7										
8										

Pair 7



Pair 8



Is it enough that two figures have a constant scale factor for corresponding sides to make them similar? ___ Explain your answer.

What can you say is needed for two figures to be similar?

Using graph paper and a ruler for accuracy, draw a pair of **similar** figures; draw a pair of **nonsimilar** figures with the **same differences in the lengths of the corresponding sides**. Have a classmate measure the figures and complete a chart like the one above for your figures and see if the results you found hold.

What if the figures are **regular polygons** (squares, triangles with all sides the same length, or other polygons with all sides the same length and all angles the same size)? Can you make a pair of regular polygons (with the same number of sides in each) that are not similar? Explain.

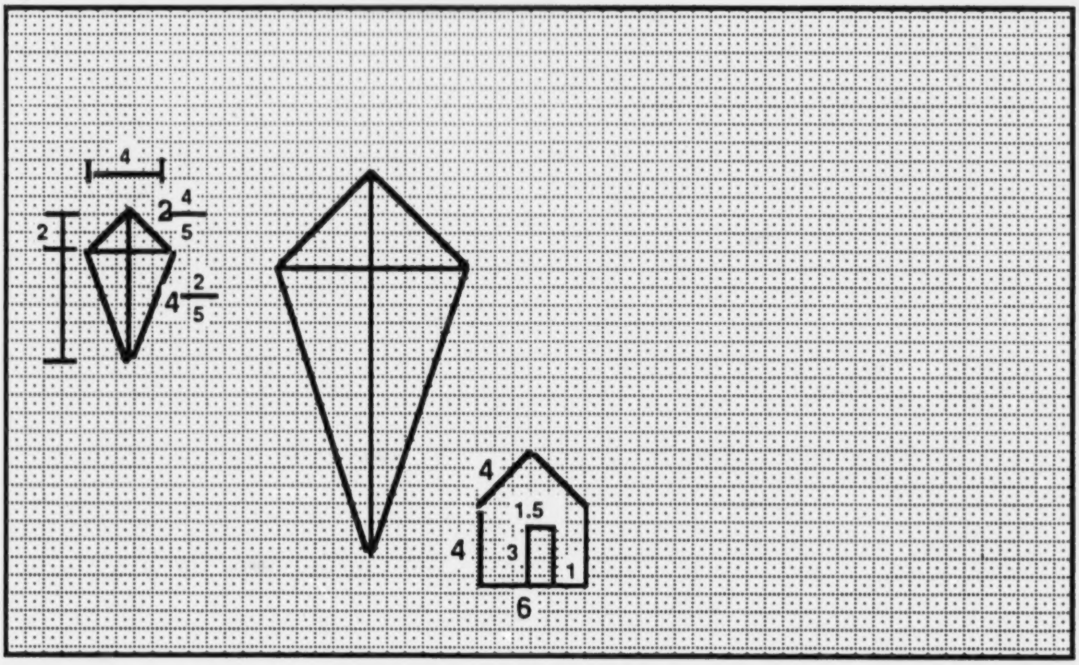
____

____

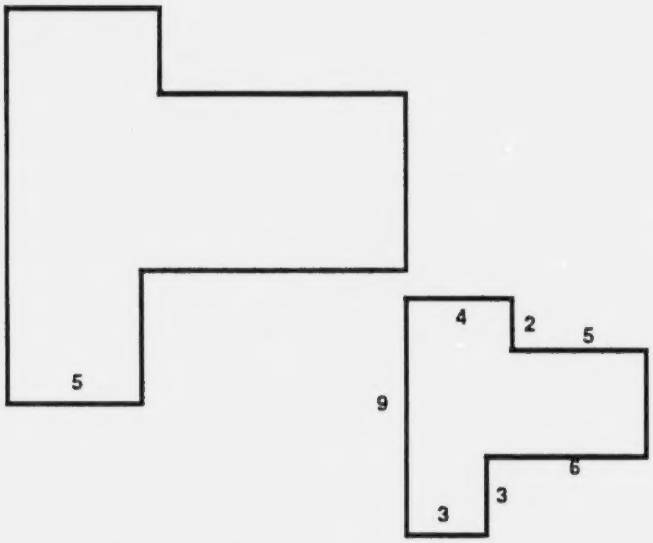
1. On the drawing that follows, the lengths are given in some unknown system. Write the lengths of the corresponding sides for the larger **kite** if its scale factor is $2\frac{1}{2}$; this means the quotient of the lengths of corresponding sides will be $2\frac{1}{2}$.

(Length in larger figure) = $2\frac{1}{2}$ x (Corresponding length in smaller figure)

2. Draw a larger **house** similar to the one pictured but with scale factor $1\frac{1}{2}$. Label the lengths of the new dimensions.



3. Find the lengths of the sides of the larger figure below **without measuring**. Pretend the figures are similar. You can see from the drawings that for every 3 units in the smaller figure, there are __ in the larger. What is the scale factor? ____
Write the lengths on the new figure. Explain how you reached your answer.



Monster Scale Factor

4. Use the **height** of your **drawing** of your monster and the monster's **full-size height** to determine the **scale factor** relating the two. Use the same unit of measurement for both. ______
Write an equation for relating measurements in the drawing and in the full-size monster using the scale factor.

5. Suppose Hollywood decides to make a movie about your monster, and plans to use a model monster two feet tall in the movie. Use the life-size height and the two-foot height of the model to decide the scale factor to use for everything on the movie set so the props will look "right."

6. "The mighty ape in the 1933 film 'King Kong' appeared to be a terrifying 50 **feet** tall; in reality, the model used was a mere 18 **inches** from head to toe." If the studio has just found some of the props from that movie to use with your monster, what size model of your monster should be used so that everything will seem to be the right size, and so your monster will appear to be the height you mean for it to be? Explain.

During this lesson, you will be asked to use all **three sizes** of your monster:

1) The scale drawing you first made on graph paper;
2) The full size of your monster;
3) The size you just found--the size the movie-model monster would need to be to use the King Kong movie props.

7. Two classmates created carnivorous (meat-eating) monsters with big appetites that depend only on their heights. One monster is 20 feet tall (life-size) and the other is 40 feet tall. If the 20-foot monster eats 4 (choose something you think it would like) __________ a day, how many will the larger one probably eat? Explain.

How many of these would a 50-foot tall monster eat? _________
Why?

How many would a 10-foot monster eat? __________
How about a 15-foot monster? ___________ Explain your answer.

AREA

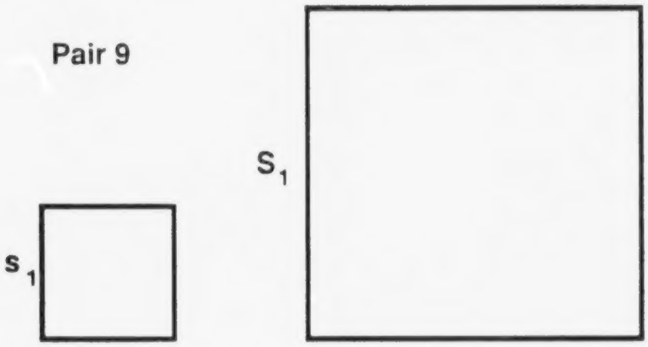
You are going to calculate the blanket sizes for both your full-size monster the movie model of your monster. To do this, first determine the blanket area needed to cover the monster on your scale drawing. Before you can find the other two blanket sizes, you will investigate the relationship between scale factor and area in the following section.

Use the pairs of figures indicated in the table that follows. Some are from the previous similar pairs (Pairs 2 and 6), and two new pairs appear following the table. You will need to make some **more measurements** to find the areas **in Pair 6**. Measure the indicated lengths in Pairs 9 and 10 to find their areas. Find the relationship between scale factor and area for similar figures.

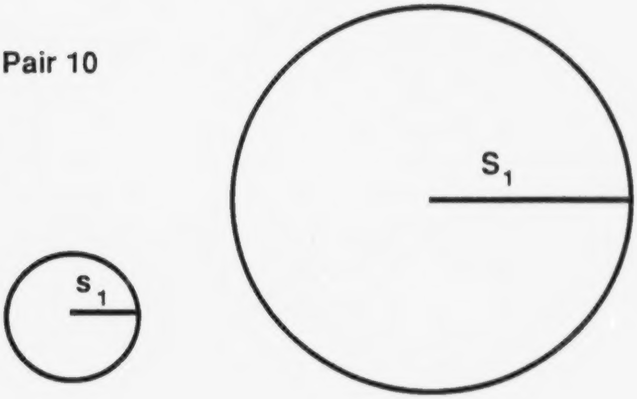
Areas of Similar Figures

Pair	s_1 (cm)	S_1 (cm)	Scale Factor $S_1 \div s_1$	area small figure	AREA Large Figure	AREA ÷ area	(Scale Factor)^2
2							
6							
9							
10							

Pair 9



Pair 10



8. Explain how the ratio of the areas of the similar figures in each pair is related to the scale factor for the figures (the ratio of the lengths of their sides).

9. You can use the relationship between scale factor and area to determine the amount of fabric needed for blankets for the movie model monster and for the full-sized monster.

a) Use the scale factor relating the scale drawing of your monster and the model size to find the area of a blanket that would cover the model. Estimate the amount of fabric that will be needed to make this blanket.

b) About how much fabric would you need to make a blanket for the full-sized monster? _____ How did you find the answer?

10. The skin covering a man weighs about 6 pounds. Estimate about how much a similar skin covering your full-size monster would weigh:

Use the scale factor of the heights:

Use the scale factor for the areas:

Use the weight of the skin covering the man:

About how much would the monster's skin weigh?

VOLUME

Using wooden cubes 1 unit on each side, make the following sized cubes and compare each to your 1 x 1 x 1 cube. Record the information in the table. To compare the volumes, count the unit cubes you use to make the larger cubes. What do you think the table values should be for an n x n x n cube?

Cube	s_1	S_1	Scale Factor $S_1 \div s_1$	volume small figure	VOLUME Large Figure	VOLUME ÷ volume	(Scale Factor)^3
2 x 2 x 2	1	2	2	1	8	8	8
3 x 3 x 3	1			1			
4 x 4 x 4	1			1			
n x n x n	1			1			

11. Use the information in this table to explain how the volumes of similiar figures are related to the scale factor for the sides.

12. How many times as tall as an average human is your monster? If this is the scale factor for comparing your monster to a human, how many times as great would the volume of your monster be compared to that of a human? How many times as great would your monster's weight be as that of an average human? Explain.

13. A human thigh bone can support up to only ten times the weight of a six-foot person. Do you think it is possible for a creature the height and weight of your monster to exist? Could any skeleton support its weight? Explain your answer.

14. Since you have found out how weight and volume are related to height, do you think it is realistic to decide on the number of victims a monster would eat each day the way we did it in Part 1? If he is twice as tall, would he eat twice as many? ______ Explain.

15. If you are female, do Part a) below; if male, do Part b).

a) The tallest known woman, Jane Munford (1895-1922), an Englishwoman, was 7 feet 11 inches. Compare her weight with yours if you had similar builds.

The shortest height recorded for an adult female was 1 foot 11 inches for Pauline Musters (1876-1895), a Dutch woman. What would you expect her weight to be, based on your height and weight?

Explain your solutions.

b) The tallest reliably measured male was Robert Wadlow (1918-1940) of the USA. He was 8 feet 11 inches. How would you expect his weight to compare with yours if you had similar body types?

The shortest recorded adult male was Calvin Phillips (1791-1812), also from the USA. His height was 2 feet $2\frac{1}{2}$ inches. About how much would you expect him to weigh?

Explain your solutions.

NEW CLOTHES

FOCUS: Meaning-Centered Lesson for Multiplication
- Making choices (Cartesian Product)

PURPOSE: The students will...
- Experience making-choices problems from a real-life situation;
- Use diagrams to represent their solutions; and
- Express solutions in written and oral presentations.

MATERIALS: NEW CLOTHES worksheet

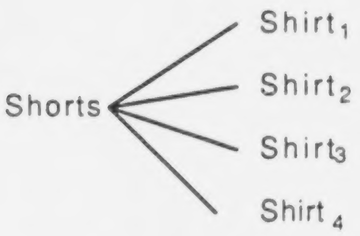
LESSON DEVELOPMENT: The worksheet may be done as individual or group work.

Make sure students understand what constitutes a different outfit.

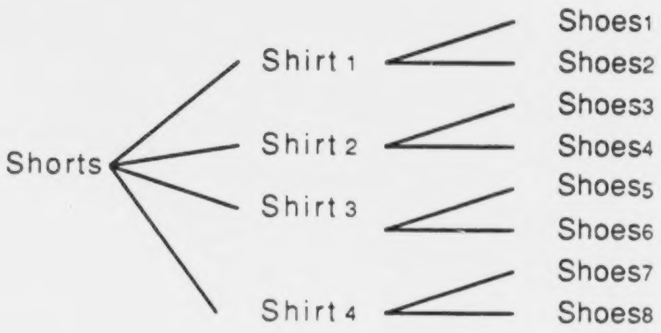
Have students share with the class the different ways of drawing or diagramming their solutions.

EXTENSION: Ask students to create their own problem involving a making- choices situation. Problems can be exchanged in class to solve and then presented to the whole class.

ANSWERS:
1. Answers may vary
2. Drawings may vary. Possible diagram:



3. Drawings may vary. Possible diagram:


Shorts
Shirt 1
Shirt 2
Shirt 3
Shirt 4
Shoes1
Shoes2
Shoes3
Shoes4
Shoes5
Shoes6
Shoes7
Shoes8

4. Multiply together the number of each item of clothing to get the total number of possible outfits.
5. 3 x 4 x 2 = 24 outfits; drawings may vary
6. Answers will vary

NEW CLOTHES

Your wardrobe really needs some sprucing up. Luckily your parents agree, so they give you some extra money and allow you to spend some of your allowance on some new clothes!

Your favorite clothing store is having a sale on their mix-and-match line of clothing. What a deal! You are going to save money and stretch your wardrobe by buying clothes that can be interchanged with each other.

You were able to buy 3 pairs of pants, 4 shirts, 1 pair of shorts, and 2 pairs of shoes.

When you got home, you spread all your new clothes on your bed and tried to figure out just how many outfits you could make.

1. Estimate how many outfits you could make with all your pants, shirts, and shoes.

____

2. On another sheet of paper, draw a picture showing how many outfits could be made with your 4 shirts and 1 pair of shorts.

3. Draw a picture showing how many outfits could be made with your 4 shirts, 1 pair of shorts, and 2 pairs of shoes.

4. Can you think of another way to answer Problems 1 and 2 without making a drawing? Explain: ____________________

____

____

____

5. How many outfits can be made with your 3 pairs of pants, 4 shirts, and 2 pairs of shoes? Write an equation and make a drawing to represent your answer. Is your answer close to your estimate in Problem 1 ?

HOMEWORK:

6. If all your favorite clothes and shoes in your closet at home really did mix and match with each other, how many outfits would you have? Draw a picture to represent your wardrobe and all the different combinations.

DECISIONS, DECISIONS

FOCUS: Application Activity for Multiplication

- Making choices (Cartesian product and fundamental counting principle)

PURPOSE: The students will...

- Apply their understanding of Cartesian products and the fundamental counting principal in solving word problems;
- Make drawings or diagrams to justify their answers;
- Deal with extraneous information;
- Write their own word problems to show their comprehension; and
- Share and discuss methods of solution.

STUDENT BACKGROUND: The students should have experience with previous meaning-centered lessons dealing with the Cartesian product and the fundamental counting principle.

MATERIALS: DECISIONS, DECISIONS worksheet

LESSON DEVELOPMENT: The worksheet may be done individually or in small groups.

In Problem 1, make sure students understand that not selecting a sauce, or not selecting a "vegetable," is also a choice.

In Problem 3, students need to realize that a double scoop could be made up of the same flavor, and that chocolate on top of chocolate, for example, is the same as chocolate on top of chocolate; do not count it twice. However, you will need a class discussion to consider if vanilla on top of chocolate is the same as chocolate on top vanilla. Perhaps, have different students work the problem both ways to show the difference in the number of resulting combinations.

EXTENSION: Ask students to create their own word problems using the Cartesian product and the fundamental counting principal. They may exchange their problems, solve, and then present their solutions to the rest of the class. Be sure to point out the different methods of solution used by the students, but also how multiplication is used in all problems.

ANSWERS:

1. 2 x 4 x 5 = 40
2. 2 x 26 = 52
3. 26 x 26 x 2 = 1152 (when chocolate on top of chocolate is considered different from vanilla on top of chocolate)
4. 4 x 3 x 2 x 1 = 24

DECISIONS, DECISIONS

Write an equation to solve each of the following problems AND justify your answers by making a drawing, diagram, or organized list on another sheet of paper.

1. "I'll gladly pay you Tuesday for a hamburger today." Wimpy was starving for a hamburger. Even though he didn't have the money yet to buy a hamburger, he was studying the menu at his favorite hamburger stand. Hamburgers could be ordered as follows. The bun may be white or wheat; you have a choice of one sauce: catsup, mustard, mayonnaise, or no sauce at all; and the hamburger may be served with one of the following: pickles, onions, tomatoes, spinach (guess who owns the hamburger stand?), or none of these. How many times could Wimpy eat at this hamburger stand without eating the same hamburger twice?

2. A new ice-cream store advertises 26 flavors of ice-cream. A single scoop costs $1.29 and a double scoop costs $1.89. They offer a regular cone and a sugar cone. How many different kinds of single scoop ice-cream cones can you order, taking into account the two kinds of cones? (Your drawing, diagram, or list needs to include only four flavors of ice-cream).

3. From the situation in Problem 2, how many different kinds of double scoop ice-cream cones can you order?

4. As soon as math class is over, Anita is planning to comb her hair in the restroom, visit two friends at their lockers (which are in different hall ways), and go to her own locker all before her English class which is right next door to her math class. She plans to do all this in five minutes! In how many ways can Anita put in order all her stops between her math and English classes? (Do you think she will be late for her English class?)

AUTO OPTIONS

FOCUS: Application Activity for Multiplication
- Making choices (Cartesian product)

PURPOSE: The student will...
- Experience a Cartesian product real-life situation and
- Make diagrams to illustrate the meaning of Cartesian products.

STUDENT BACKGROUND: Students need to be familiar with the Meaning-Centered Lesson for Multiplication on Making Choices, New Clothes.

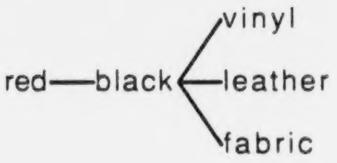
MATERIALS: AUTO OPTIONS worksheet

LESSON DEVELOPMENT: Some students may argue that some color combinations are ridiculous. For the purpose of the lesson, ask students to be open to all combinations.

EXTENSION: Have the students make a chart of one course at your school, the teachers who teach it, and the periods it is offered. Find out how many combinations are possible. Repeat this for each class.

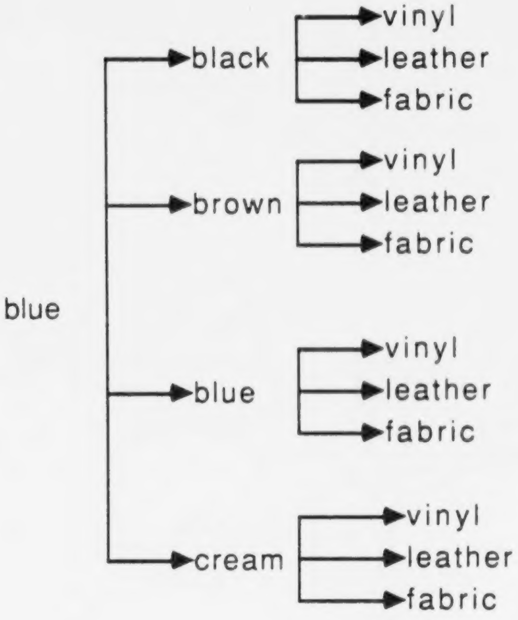
ANSWERS:

1.


red
black
vinyl
leather
fabric

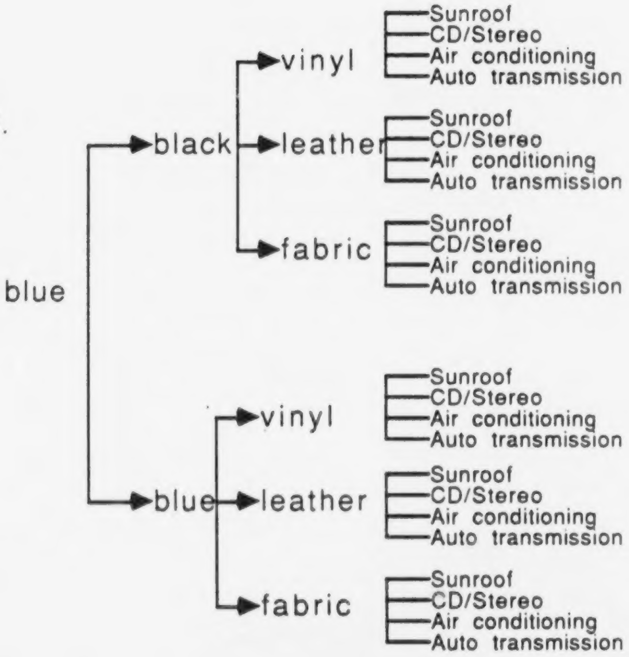
; 3 combinations;
1 x 1 x 3 = 3

2.


blue
black
vinyl
leather
fabric
brown
vinyl
leather
fabric
blue
vinyl
leather
fabric
cream
vinyl
leather
fabric

; 15 combinations;
1 x 5 x 3 = 15

3.


blue
black
vinyl
Sunroof
CD/Stereo
Air conditioning
Auto transmission
leather
Sunroof
CD/Stereo
Air conditioning
Auto transmission
fabric
Sunroof
CD/Stereo
Air conditioning
Auto transmission
blue
vinyl
Sunroof
CD/Stereo
Air conditioning
Auto transmission
leather
Sunroof
CD/Stereo
Air conditioning
Auto transmission
fabric
Sunroof
CD/Stereo
Air conditioning
Auto transmission

; 60 combinations
4 x 1 x 5 x 3 = 60

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4. Estimates will vary; 360 combinations; 4 x 6 x 5 x 3 = 360

5.

OPTIONS	EXT COLOR	INT COLOR	SEAT COVER
1	6	5	2

1 x 6 x 5 x 2 = 60; 60 combinations

6.

OPTIONS	EXT COLOR	INT COLOR	SEAT COVER
2	2	2	3

2 x 2 x 2 x 3 = 24; 24 combinations.

AUTO OPTIONS

OPTIONS	EXTERIOR COLORS	INTERIOR COLORS	SEAT COVER
Sunroof $1200	red	black	vinyl
Stereo $500 or CD $900	white	blue	fabric
Air conditioning $1200	black	cream	leather $950
Automatic Transmission $700	grey	grey	
(all other features come standard with car)	blue		

You are in the market for a new car and after weeks of looking at and test driving many makes and models, you make a decision. However, you must now decide on the available options, the exterior and interior colors, and type of seat cover.

1. If you decide you want a red exterior, black interior, and no options, how many combinations with seat covers are possible? Make a diagram of the combinations.

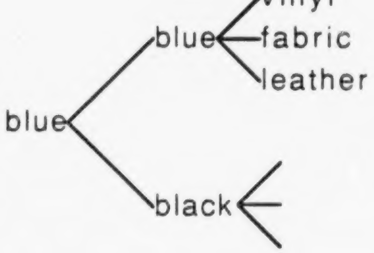
EXT COLOR	INT COLOR	SEAT COVER
red (1)	black (1)	3

Write an equation and solve.

2. If you decide you want a blue exterior and have no other preferences, how many combinations are possible considering only interior colors
(even if they may be strange combinations) and seat cover?

EXT COLOR	INT COLOR	SEAT COVER
blue (1)	5	3

Complete the diagram for all combinations.

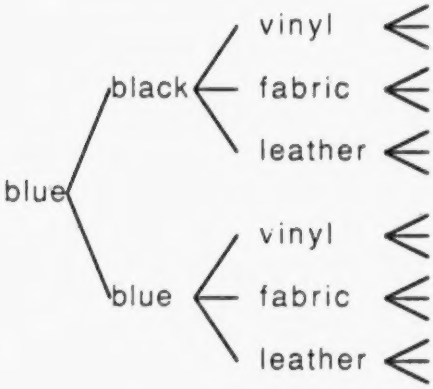

blue
blue
vinyl
fabric
leather
black

Write an equation and solve.

3. Now, also consider the options. You can afford only one option. How many combinations are possible?

OPTIONS	EXT COLOR	INT COLOR	SEAT COVER
4	blue(1)	5	3

Complete the diagram.


blue
black
vinyl
fabric
leather
blue
vinyl
fabric
leather

Write an equation and solve.

4. How many combinations are possible considering all your choices? Make an estimate.______________________

OPTIONS	EXT COLOR	INT COLOR	SEAT COVER
4	6	5	3

Write an equation and solve.

5. You decide the only option you absolutely have to have is a stereo or CD, and you most definitely do not want leather seats (too expensive). Now, how many combinations are possible?

Fill in totals.

OPTIONS	EXT COLOR	INT COLOR	SEAT COVERS
______	______	______	_______

Write an equation and solve.

6. You are willing to pay up to $900 in options, and have narrowed down your color preferences to either red or gray for exterior colors and either black or blue for interior colors. How many combinations are possible?

Fill in the totals.

OPTIONS	EXT COLOR	INT COLOR	SEAT COVER
______	______	______	______

Write an equation and solve.

EXCLAMATION!

FOCUS: Meaning-Centered Lesson for Multiplication
- Making choices (fundamental counting principle)

PURPOSE: The students will...
- Make tree diagrams to find total arrangements possible, and
- Learn factorial notation and how it relates to finding total arrangements possible.

MATERIALS: EXCLAMATION! worksheet and calculator.

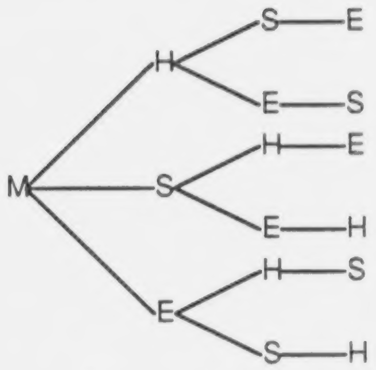
LESSON DEVELOPMENT: The worksheet may be done individually or in small groups. In Part A, after the students have worked through Problems 1 through 10, the teacher may have to help the students understand why one would multiply together the number of choices you have for each spot to get the total number of arrangements. In Part C, the teacher may need to give more examples and explanations as needed for factorial notation. In Part C, Problem 3, the teacher may want to have students share how they found the final answer, since the answer does not fit on a calculator display.

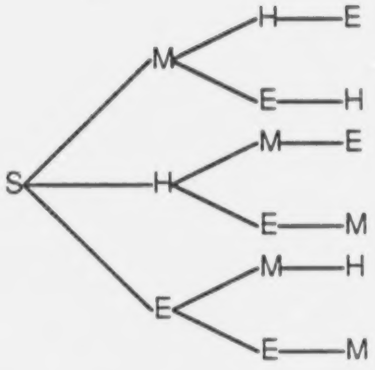
ANSWERS:

Part A

1. M; S; HMS
   S; M; HSM
   M; H; SMH
   H; M; SHM
2. 3
3. 2
4. 2
5. 2
6. 4
7. 1
8. 1
9. 6
10. 3,2,1,6
11. Answers will vary but should include the notion that multiplying the number of choices you have for each space gives you the total number of arrangements.

Part B
1.


M
H
S
E
S
E
H
E
E
H
H
S
S
H


S
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M
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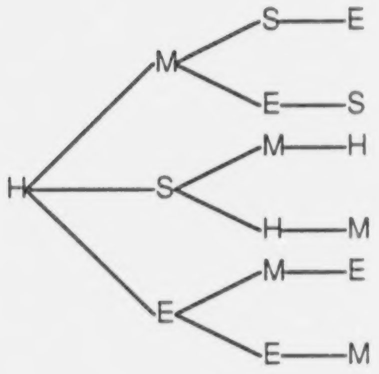
M


H
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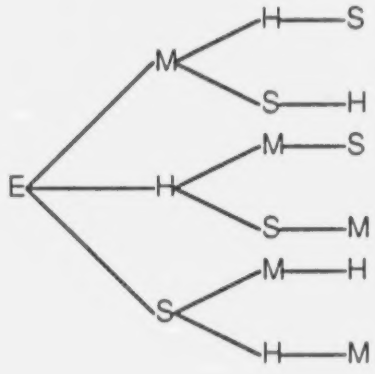
M


E
M
H
S
S
H
H
M
S
S
M
S
M
H
H
M

2. 4 x 3 x 2 x 1 = 24

Part C

1. 3,628,800
2. 5,040
3. 403,291,461,126,605,635,584,000,000

HOMEWORK: 252 days

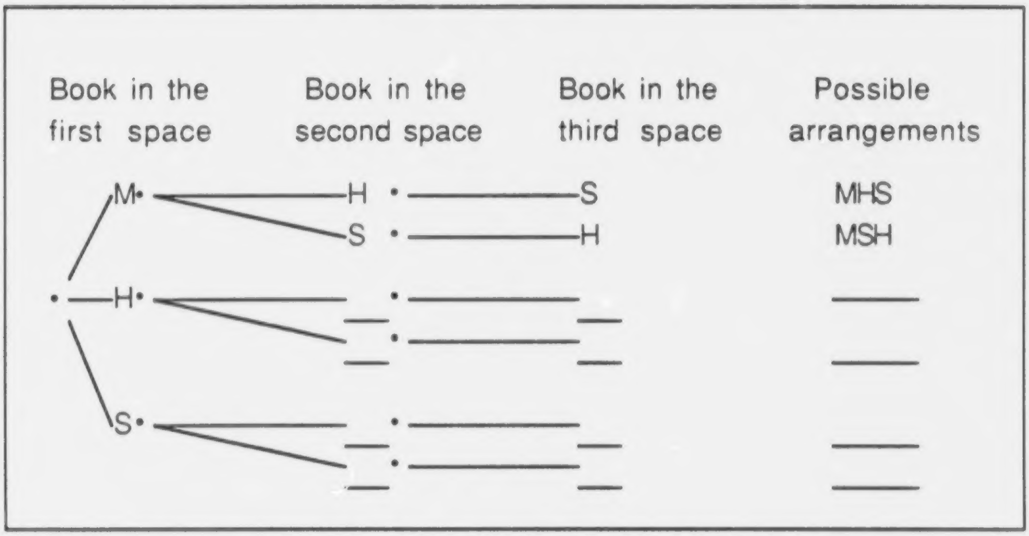
SOURCE: Adapted from Arrangements and Selections, Experiences in Mathematical Discovery. (1966). NCTM Publication

EXCLAMATION!

Part A

It was the first day of school and you already had your math, history, and science textbooks checked out to you! Luckily, your locker was already assigned to you, too. How many ways can your three books be stacked in your locker?

1. Complete this tree diagram of the different arrangements.



2. How many different books can be placed first? ____________________
3. If your math book is placed first, in how many ways can the second space be filled? ____________________
4. If your history book is placed first, in how many ways can the second space be filled? ____________________
5. If you have 1 choice for the first space and 2 choices for the second space, how many total arrangements are possible?__________________
6. If you have 2 choices for the first space and 2 choices for the second space, that makes a total of how many possible arrangements?________
7. After the first space has been filled with your math book, and the second space has been filled with your history book, in how many ways can the third space be filled? ____________________
8. In general, after the first and second spaces have been filled, in how many ways can the third space be filled? ____________________
9. If you have 3 choices for the first space, 2 choices for the second space, and 1 choice for the third space, how many total arrangements are possible? ____________________

10. Complete this chart:

Number of ways first space can be filled	Number of Ways second space can be filled if first space is filled	Number of ways third space can be filled if first and second	Number of ways books can be arranged
________	________	________	________

11. Explain in writing the meaning of the above chart and what the relationship is between the first three numbers and the last number.

Part B

The second day of school is over and your English teacher handed out books today. Now you have four books.

1. On another sheet of paper, make a tree diagram to show the possible arrangements your books could have in your locker.
2. Also use the method you used in Problem 10, Part A, to find your answer. Your answers to Problems 1 and 2 should be the same. If not, go back and find your error.

Part C

Expressions such as $3 \times 2 \times 1$ and $4 \times 3 \times 2 \times 1$ can be abbreviated by special notation called "factorial", using the factorial symbol " ! ". In general, the number of possible arrangements of n things is n! (read, "n factorial").

n! is the product of the first n counting numbers.

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$1! = 1$

Write equations using factorial notation and solve the following problems.

1. The yearbook staff is trying to take a picture of the four ASB officers and their six assistants. The staff is trying all arrangements to get the best picture. How many arrangements are possible if they must stand in a row?________________
2. In how many ways can seven people line up to buy tickets for the next school dance?________________________________

3. The twenty-six letters of the alphabet can be arranged 26! different ways. What number is this? Can you read your answer? See if you can find the names of each of those place value columns! ________________________________

HOMEWORK: In Problem 1, Part C, how long would it take the yearbook staff to make all those arrangements of the ten ASB students if it takes six seconds for each arrangement and they work non-stop?

HIT BY A 10-TON TRUCK

FOCUS: Application Activity for Multiplication

- Formulas using multiplication
- Rate

PURPOSE: The student will ...

- Utilize rate conversions in dealing with a scientific formula, and
- Develop some feel for the magnitude of the momentum of a moving car or truck.

STUDENT BACKGROUND: The student should be familiar with square root and with rate conversions (km/h --> m/s [metric] or mi./hr. --> ft./sec. [English]). Some familiarity with mass and velocity (or speed) in metric units is also assumed.

TEACHER BACKGROUND: The science teacher might be interested in working on this topic with you. The "momentum" of an object has an exact scientific meaning: (momentum of object) = (mass m of object) x (its speed or velocity v). Part of this lesson involves the velocity v of an object dropped from a height h (ignoring air resistance): $v^2 = 2gh$ or $v = \sqrt{2gh}$, where g is the acceleration due to gravity. Hence, (momentum of dropped object) = (its mass) x $\sqrt{2gh}$. With metric units (g = 9.8 m/s, h in meters), so $\sqrt{2gh} = 4.4\sqrt{h}$. There is no special name for momentum units. [In English units, where g = 32 ft./sec^2, the radical simplifies nicely to $8\sqrt{h}$. Unfortunately, mass is not measured in pounds but in a unit called a slug (!), hardly familiar to the students.]

MATERIALS: 0.5 kg and 1 kg masses (or 1 lb. and 2 lb. weights) [e.g., bags of sand or beans], meter stick (yardstick or measuring tape), calculators.

LESSON DEVELOPMENT: Ask, "Has anyone ever been hit by a truck or a car? How does it feel?" After whatever discussion results, ask what "momentum" means (the term is used often in sports but in a nontechnical sense). Explain that momentum is actually a scientific term and is defined by

(momentum of object) = (mass m of object) x (its speed or velocity v), or
(momentum of object) = m x v.

Data Collection. Explain that to get a feel for the size of momentum, the class will drop (not throw) known masses and let them hit their hands. Since things fall so fast that it is difficult to measure their speeds, you will use a formula for v: $v = 4.4\sqrt{h}$, where h is measured in meters and v in m/s. (The English version is $v = 8\sqrt{h}$, with h in feet and v in ft./sec.) The only measurements that are needed are the height h of the drop (how far it is to the hand from where the mass is dropped), in meters, and the mass m of the object, in kilograms, and you can use the formula,

(momentum of dropped object) = m x $4.4\sqrt{h}$ [or m x $8\sqrt{h}$), English].

Have volunteers or groups collect data, using different values of h and calculating the momentum in each case.

The Main Question. Return to the main question, phrased in terms of momentum: What is the momentum of a truck? The general formula,

(momentum) = m x v (m in kg, v in m/s),

still applies. Have pairs or small groups choose some combination of vehicle and speed, calculate the momentum, and then compare that figure with the one from the small mass. These are fairly realistic data: mass of car = 1000 kg (2200 lbs.), of small truck = 2000 kg, of larger truck = 4000 kg, of a "10-ton truck" = 9100 kg. So that students choose a reasonable speed, remind them that 55 mi./hr. is about 90 km/h.

Here are some samples:

1000 kg car, 90 km/h (= 25 m/s): momentum = 25,000 momentum units

1 kg mass, dropped from 1 m: momentum = 4.4 momentum units

The car has more than 5000 times the momentum of the 1 kg mass (looked at another way, dropping a 5000+ kg mass from 1 m would give the same momentum).

EXTENSION: Sports enthusiasts might like to calculate momentums for athletes. They may know figures; here are some: champion dash-man--90 kg, 100 m/10 s; champion dash-woman--60 kg, 100 m/11 s; a fast pro football player--100 kg, 40 m/5 s.

IT'S ONLY FAIR

FOCUS: Meaning-Centered Lesson for Division
- Sharing equally
- Repeated subtraction

PURPOSE: The student will ...
- Gain a deeper understanding of the two fundamental types of division word problems;
- Learn to recognize sharing-equally and repeated-subtraction situations; and
- Interpret results in repeated subtraction situations.

STUDENT BACKGROUND: If calculators are not available, students should be able to multiply and divide decimals, and to use decimal number sense.

MATERIALS: IT'S ONLY FAIR worksheet

LESSON DEVELOPMENT: It is crucial that the students take the time to understand the two situations by drawing the pictures and doing the repeated subtraction. In 5 b-c, you may have to show how to get the information from either the hand calculation or the calculator results. It may be beneficial when identifying the two situations in #6 to have the students justify their responses using appropriate language or drawings and share/discuss with the class.

ANSWERS: 2. $26 \div 4$, $6\frac{1}{2}$ inches 3. \$1.81; 2.50 - .69 = \$1.81. 4. \$1.12. 5. a) 3 bags, $2.50 \div .69$ or 2.50- .69 - .69 - .69; b) division; c) yes, 43¢. 6. a) sharing equally; b) repeated subtraction; c) repeated subtraction; d) sharing equally; e) sharing equally; f) repeated subtraction; g) repeated subtraction; h) sharing equally; i) repeated subtraction; j) sharing equally.

IT'S ONLY FAIR

Four friends buy a giant string of licorice 26 inches long. If they share the cost equally, about how many inches of licorice does each person get?

1. Draw a picture to show what needs to be done.

2. Mathematically, how would you solve this word problem? Find the answer.

** This division word problem is a **sharing-equally situation** since a total (26 inches) is shared equally among a number of persons (4).

Betty Lou decides to return to the candy shop and buy more licorice. She has $2.50 to spend and the licorice sells for 69¢ a bag.

3. If Betty Lou buys only one bag, how much money does she have left?
Mathematically, write an expression to show this.

4. If Betty Lou buys two bags of licorice, how much money does she have left?

One way you can show this is by subtracting 69¢ twice:
$2.50 - 69¢ - 69¢ = money left.

5. a) How many bags of licorice can she buy with $2.50?

   b) Which operation allows you to find the answer in one step?

   c) Will Betty Lou have any money left? How much?

** This division word problem is a **repeated-subtraction situation** since you are finding how many equal amounts (69¢) in the total ($2.50) and this can be done by subtracting repeatedly--but more efficiently by dividing.

6. Identify each of the following division word problems as either **sharing-equally situations** or **repeated-subtraction situations**.

a) You buy a cassette tape player for $225.00 and plan to make three equal payments. How much is each payment?

____

b) Baseball cards are 35¢ a pack. You have $2.50. How many packs can you buy? __________________________________

c) Rides at the State Fair are 75¢ each. You have $10. How many rides can you go on? ____________________________

d) You and your friend go out to dinner. The bill comes to $26.86 and you will split the check equally. How much do you pay?

____

e) Sally, Anita, and Jack earned a total of $24.96 washing cars. On the average, how much did each person earn?______________________________

f) A full bus holds 43 children. If 119 children go on a field trip, how many buses are needed?

____

g) Marguerite uses $26 worth of art supplies to make one painting. How many paintings can she make for $234?

____

h) The distance between Seattle and Denver is 2210 km. If you want to reach Denver in 3 days, how many kilometers must you drive each day?

____

i) The distance between Dallas and Atlanta is 1323 km. If you drive 500 km a day, how many days will it take you to get to Atlanta?

____

____

j) You have 60 minutes to do five assignments. On the average,. how long should each assignment take you?__________________________

EXTENSION: Go back and solve two word problems of each type. Explain how to handle the remainder in each problem (if there is a remainder)

EASY AS PIE

FOCUS: Meaning-Centered Lesson for Division

- Sharing equally
- Repeated subtraction

PURPOSE: The student will...

- Gain a deeper understanding of the two fundamental types of division word problems;
- Gain experience dividing fractions in a real-life setting;
- Use picture representations to understand division of fractions;
- Learn to recognize sharing-equally and repeated-subtraction situations;
- Estimate fractional parts on a drawing; and
- Interpret fractional remainders.

STUDENT BACKGROUND: Students should be operationally competent with fractions, and should possess fraction number sense. Students need to be familiar with vocabulary and concepts of the previous Meaning Centered Lesson for Division, It's Only Fair.

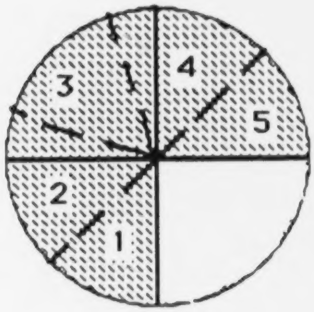
MATERIALS: EASY AS PIE worksheet, colored pencils(optional).

LESSON DEVELOPMENT: Since students already know how to divide; the emphasis is on the different basic meanings for division. Do problems #1 and #2 as a teacher-directed whole group activity. Estimating may be difficult for some students; make sure they use erasable materials. The closer the estimate the better the connection will be between the answer and the concept of division of fractions. Their drawings and estimates will also aid in their interpretations of the fractional remainders. Review division of fractions, and inverse operations. Students may choose to multiply by a fraction instead of dividing by a whole number ($\frac{3}{4} \div 5$ or $\frac{3}{4} \times \frac{1}{5}$). Keep the focus on division, but reassure them that both are acceptable. When showing ways to verify answers, students should realize that multiplication with a whole number can be interpreted as repeated addition. Encourage students to use appropriate language in their explanations.

EXTENSION: Have students work in small groups. Exchange word problems and solve. Share examples and solutions with class.

ANSWERS:

1a&b)

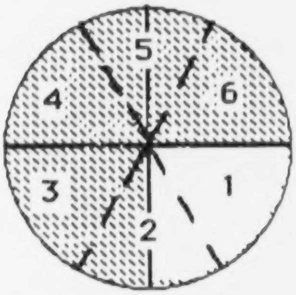


c) $\frac{3}{20}$ of a pie. d) $5 \times \frac{3}{20} = \frac{3}{4}$;

$\frac{3}{20} + \frac{3}{20} + \frac{3}{20} + \frac{3}{20} + \frac{3}{20} = \frac{3}{4}$ or

$\frac{3}{4} - \frac{3}{20} - \frac{3}{20} - \frac{3}{20} - \frac{3}{20} - \frac{3}{20} = 0$

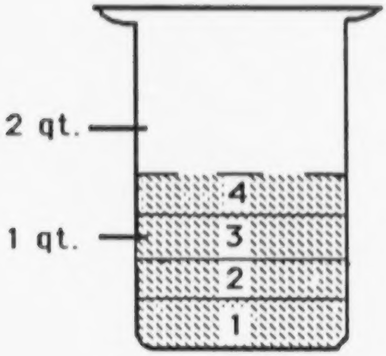
2a&b)



c) 4 full servings. d) Yes; $\frac{3}{4} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6}$.

e) Yes; $\frac{1}{2}$ of $\frac{1}{6}$.

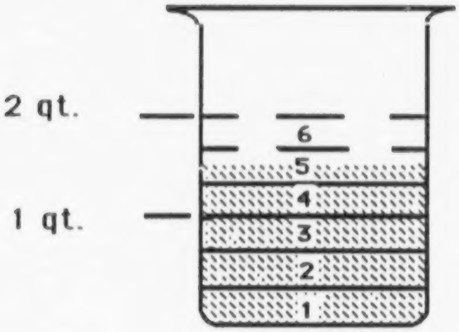
3a&b)



c) $\frac{3}{8}$ qt.

d) $\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}=1\frac{1}{2}$ or $1\frac{1}{2}-\frac{3}{8}-\frac{3}{8}-\frac{3}{8}-\frac{3}{8}=0$ or $4\times\frac{3}{8}$ e) sharing equally; explanations will vary.

4a&b)



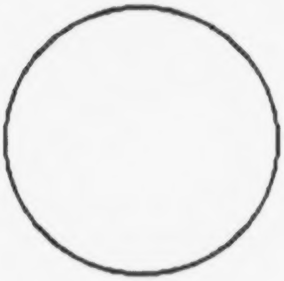
c) 4 full servings. d) Yes; $1\frac{1}{2}-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}=\frac{1}{6}$. e) Yes; $\frac{1}{2}$ 0f $\frac{1}{3}$.

f) repeated subtraction; explanations will vary.

EASY AS PIE

$\frac{3}{4}$ of an apple pie needs to be shared among 5 people.

1a) Lightly shade $\frac{3}{4}$ of the pie.



b) Using dashed lines, estimate how the $\frac{3}{4}$ pie will be split into 5 equal pieces, numbering 1 through 5.

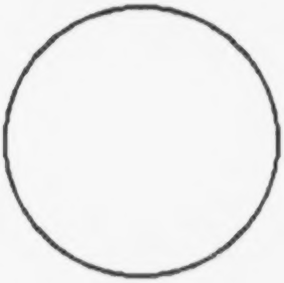
c) What fraction of the whole pie does each person get?

d) Knowing the answer, show two ways (using two different operations) to check it.

** This division word problem is a **sharing-equally situation** since the total ($\frac{3}{4}$ of pie) is shared equally (among 5 people).

2. Using $\frac{3}{4}$ of another pie, you decide to make each serving the size of $\frac{1}{6}$ of the whole pie.

a) Lightly shade $\frac{3}{4}$ of the pie.



b) Using dashed lines, estimate how the whole pie will be split into 6 equal pieces, numbering each piece 1 through 6.

c) How many full servings (the size of $\frac{1}{6}$ of the whole pie) can you get out of $\frac{3}{4}$ of the pie?

d) Does your drawing support your answer? How can you check your answer using another operation?

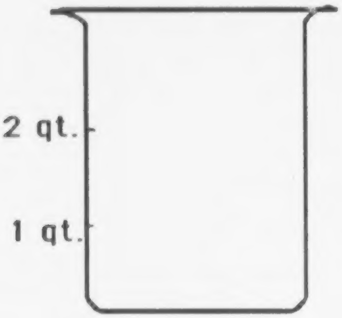
e) Will you have a piece smaller than $\frac{1}{6}$ of pie left over?

The leftover is ________________ of $\frac{1}{6}$.

** This division word problem is a **repeated-subtraction situation** since you are finding how many equal parts (the size of $\frac{1}{6}$ of a pie) in the total ($\frac{3}{4}$ of a pie) and this could be done by subtracting repeatedly.

3. The pitcher contains 1 $\frac{1}{2}$ quarts of lemonade.

a) Lightly shade 1 $\frac{1}{2}$ qts.



b) You and 3 friends play volleyball and become very thirsty. Using dashed lines, estimate how 1 $\frac{1}{2}$ qt of lemonade will be split into 4 equal parts, numbering each part 1 through 4.

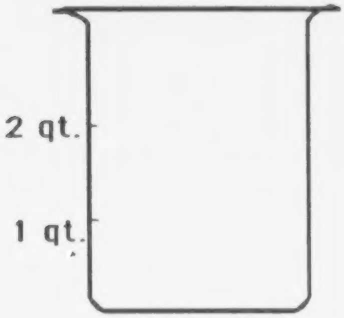
c) How much lemonade will each person get? ____________qt.

d) Knowing the answer, show two ways (using two different operations) to check it.

e) This division word problem is which situation, **sharing equally** or **repeated subtraction**? Explain.

4. The next day you make another pitcher of lemonade (1 $\frac{1}{2}$ qt.), and decide to make each serving $\frac{1}{3}$ qt.

a) Lightly shade $1\frac{1}{2}$ qt.



b) Using dashed lines, mark off equal parts (of $\frac{1}{3}$ qt. each), numbering 1,2,3,....

c) How many full servings ($\frac{1}{3}$ qt.) of lemonade can you get out of $1\frac{1}{2}$ qt.?

d) Does your drawing support this answer? How can you prove it using another operation?

e) Will you have some lemonade left over?

The leftover is ___________ of $\frac{1}{3}$ qt.

f) This division word problem is which situation, **sharing equally** or **repeated subtraction**? Explain.

EXTENSION: A big container holds 128 ounces of caramel corn. Write two division word problems- -one sharing-equally situation and one repeated- subtraction situation. Make a drawing.

WISE SHOPPER

FOCUS: Application Activity for Division

- Sharing equally

PURPOSE: The student will...

- Experience coupon shopping as in real life;
- Understand the concept of "better buy" by considering cost per unit;
- Interpret results;
- Learn to think more critically about all factors involved in making a wise buy; and
- Express himself/herself in writing about what they have learned in this lesson.

STUDENT BACKGROUND: Students should be able to round decimals.

MATERIALS: WISE SHOPPER worksheet and calculators.

LESSON DEVELOPMENT: It may be beneficial to have the actual items in the two different sizes as visual aids for students. This lesson can be done in small groups where each group would have different items and would then report its conclusions to the class. Money figures when calculating cost per ounce will need to be rounded to the nearest cent, and students may need help interpreting their results. Students may use proportions where others will divide to solve. Have students share their work with the class. Depending on what work you have done with rates, you might note that value per weight unit is a special rate.

ANSWERS:

1. Detergent: $2.69 for 64 oz. or $1.13 for 39 oz.
   Peanut Butter: $3.23 for 28 oz. or $1.85 for 18 oz.
   Pancake Syrup: $2.11 for 24 oz. or $1.34 for 12 oz.
2. Detergent: $0.04 (64 oz.), $0.03 (39 oz.); Peanut Butter: $0.12 (28 oz.), $0.10 (18 oz.); Pancake Syrup: $0.09 (24 oz.), $0.11 (12 oz.)
3. Detergent: 39 oz. size; Peanut Butter: 18 oz. size; Pancake Syrup 24 oz. size.
4. Answers will vary. 5. $2.70.

EXTENSION: Detergent: 39 oz. size; Peanut Butter: 18 oz. size; Pancake Syrup: 24 oz. size.

WISE SHOPPER

Your mother hates to cut out and use coupons for groceries, but realizes the savings can be great. She offers you a deal. She tells you that if you go to the trouble of finding and using coupons of items she already buys, then the savings will go directly to you. Knowing that Supermart doubles the value of coupons, you realize you can make a lot of money. Immediately the search begins and you find three coupons for the following items: Detergent: 75¢, Peanut Butter: 25¢, Pancake Syrup: 35¢. You take off for Supermart. You are about to take the economy size (largest size) off the shelf, but you stop to consider which is the better buy.

Detergent: $4.19 for 64 ounces or $2.63 for 39 ounces.

Peanut Butter: $3.73 for 28 ounces or $2.35 for18 ounces.

Pancake Syrup: $2.81 for 24 ounces or $2.04 for 12 ounces.

Answer the following questions.

1. How much do you pay for each size of each item after the coupons are redeemed? Remember Supermart doubles coupons.

Detergent: ________________ for 64 ounces or ______________ for 39 ounces.

Peanut Butter: ______________ for 28 ounces or ____________ for 18 ounces.

Pancake Syrup: ______________ for 24 ounces or ____________ for 12 ounces.

2. How much do you pay for each ounce to the nearest cent?

Detergent: ______________ for one ounce (64 ounce size)
______________ for one ounce (39 ounce size)

Peanut Butter: ____________________ for one ounce (28 ounce size)
____________________ for one ounce (18 ounce size)

Pancake Syrup: ____________________ for one ounce (24 ounce size)
____________________ for one ounce (12 ounce size)

3. Which size of each item is a better buy?

Detergent___________Peanut Butter________________Pancake Syrup_______________

4. Is it always a smart buy on your part to buy the smaller size whenever it is the better buy? (Consider how often you need to shop and the savings each time.) ________________________________

____

____

Is the economy size always better to buy? Explain. ____________

____

____

____

5. How much money did you earn shopping for these three items?

____

EXTENSION: Would your choice of item (size) be the same if you decide to shop at Foodville where coupons are redeemed at face value (not doubled)?. Explain. Show work.

DISNEYLAND

FOCUS: Application Activity for Division

- Sharing equally
- Repeated subtraction

PURPOSE: The student will...

- Experience both types of division word problems within the same context, and
- Interpret results (money, rounding).

STUDENT BACKGROUND: If calculators are not available, students should be able to divide and multiply decimals and students should possess decimal number sense. Students should be familiar with the two types of division word problems after doing the meaning-centered lessons for division.

MATERIALS: DISNEYLAND worksheet

LESSON DEVELOPMENT: Students may want to draw diagrams to illustrate sharing-equally situations and to show the repeated subtraction. Students will need to interpret answers when finding amount of spending money and number of persons. Be sure, in a summary, to draw students' attention to the two kinds of situations that division addresses.

ANSWERS: 1. Disneyland Hotel 2. 500 - 352 = 148; 148 ÷ 3 = $49.33
3. a) 75 x 3 = 225; 500 - 225 = $275 b) The Grand Hotel
4. a) 25 x 5 = 125; 500 - 125 = 375; yes b) Hilton Hotel and Towers
5. 500 - 294 = 206; 206 ÷ 5 = $41.20 6. 500 - 389 = 111; 111 ÷ 4 = $27.75 7. a) 2 persons b) 5 persons c) $4 d) 5 candy bars; 3 cans of soda; 45¢ 8. Answers will vary.

DISNEYLAND

You win a cash grand prize in a sweepstakes and decide to treat yourself and a couple of friends (3 including yourself) to a 2-day vacation at Disneyland. You are willing to spend $500 for hotel accommodations, admission, and extras. You have found an advertisement for some "packages" (see below).

Answer the following questions.
1. From the advertisement, what are the most expensive accommodations for 3 persons?
2. How much spending money will each person get after paying for the package? Write an equation and give the answer.

3. a) If you decide you want $75 each for spending money, how much money will you have left for the package?
   b) With the remaining money, which is the most expensive hotel you can afford?

4. At the last minute you change your mind and would like to invite another friend or two.
a) Can you afford to accommodate two more friends and still have $25 each for spending money? Explain.

b) With the remaining money, which is the most expensive hotel you can afford.

5. If there are five of you, what is the greatest amount of spending money each of you can have and still afford a package? Show work.

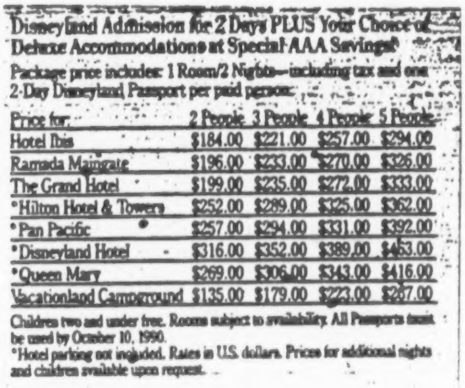
6. If there are four of you, and you decide you definitely want to stay at the Disneyland Hotel, how much spending money will each of you get? Show work.

7. You and your friends are having so much fun that you decide to come back for a third day. A one-day passport costs $26.00.
a) With a total of $69 left, how many persons can return for a third day?

b) What if you generously agree to pay 1/2 of everyone's ticket (out of emergency funds; this is an emergency); then how many persons can return for a third day?

c) How much money will be left over?
d) Since it is very little money, you decide to buy as many candy bars (35¢ each) as possible with half the money, and cans of soda (60¢ each) with the other half.
How many candy bars can you get? ____________
How many cans of soda can you get? ____________
How much money do you have left over? ____________

8. Now, it is really your turn. Decide how many friends to invite, which hotel accommodations you prefer, and how much spending money you want (not necessarily in that order). The limit is still $500. Show all work.



Disneyland Admission for 2 Days PLUS Your Choice of Deluxe Accommodations at Special AAA Savings!

Package price includes: 1 Room/2 Nights—including tax and one 2-Day Disneyland Passport per paid person.

Price for:	2 People	3 People	4 People	5 People
Hotel Ibis	$184.00	$221.00	$257.00	$294.00
Ramada Maingate	$196.00	$233.00	$270.00	$326.00
The Grand Hotel	$199.00	$235.00	$272.00	$333.00
*Hilton Hotel & Towers	$252.00	$289.00	$325.00	$362.00
*Pan Pacific	$257.00	$294.00	$331.00	$392.00
*Disneyland Hotel	$316.00	$352.00	$389.00	$453.00
*Queen Mary	$269.00	$306.00	$343.00	$416.00
Vacationland Campground	$135.00	$179.00	$223.00	$267.00

Children two and under free. Rooms subject to availability. All Passports must be used by October 10, 1990.
*Hotel parking not included. Rates in U.S. dollars. Prices for additional nights and children available upon request.

MISSING FACTORS

FOCUS: Application Activity for Division
- Missing Factor

PURPOSE: The student will . . .
- Have experience with missing factor settings.

STUDENT BACKGROUND: The student should know the formulas $d = rt$ and $I = Prt$, the formulas for the areas of rectangular and triangular regions, and the formula for the volume of a rectangular solid.

TEACHER BACKGROUND: Sometimes a student may correctly think of a missing-factor multiplication equation for what is intended as a division problem. If you have not yet given attention to solving equations with a missing factor, this lesson might be useful as an avenue to solving such equations. The settings chosen usually bring multiplication equations quickly to mind.

MATERIALS: MISSING FACTORS worksheet, calculators

LESSON DEVELOPMENT: Work through the preliminary problem, emphasizing the key idea, and giving equations like $15x = 9$ to get quick feedback. You may wish to have pairs work on problems 5-10 if your students have not worked many multi-step problems. You may need to review the formulas used in the exercises, and that there are 5280 feet in a mile (for #9).

EXTENSIONS: Similar exercises can be designed around other area or volume formulas familiar to the class, or such relationships as $C = 2\pi r$.

ANSWERS:

1. 175 m, 11.04 m^2, 1.2 m 2. 24 cu. ft., 4 1/2 ft., 0.4 m
3. 11.76 cm^2, 7 cm, 8 cm 4. 720 mi., 0.8 hr. (or 48 min.), 400 mi./hr.
5. 6 or 8 boxes, depending on how they are turned in stacking
6. \$324.6666...., so \$324.66 or \$324.67 7. \$3402
8. 30 sec. 9. 15 5/8 ft., 338 (337.92) bags 10. 7.5 cm

MISSING FACTORS

Solar panel material comes in pieces 1.2 meters wide. You want to make a rectangular solar panel with area 18 square meters. How long should your piece of panel material be?

Most people would think of $A = \ell w$, or $18 = \ell \times 1.2$. How do you find the length? $\ell = 18 \div 1.2 = 15$. **You can divide to find a missing factor.**

1. Find the missing measurements for these rectangular pieces.
   $A = 210\ m^2$, $w = 1.2\ m$, $\ell =$ ________
   $\ell = 9.2\ m$, $w = 1.2\ m$, $A =$ ________ (Is a factor missing?)
   $\ell = 30\ m$, $A = 36\ m^2$, $w =$ ________

2. Find the missing measurements for these rectangular solids, with dimensions ℓ, w, and h, and volume V.
   $\ell = 4$ ft., $w = 2$ ft., $h = 3$ ft., $V =$ ________
   $\ell = 4$ ft., $w = 3$ ft., $V = 54$ cu. ft., $h =$ ________
   $\ell = 4.5\ m$, $h = 6\ m$, $V = 10.8\ m^3$, $w =$ ________

3. Find the missing measurements for these triangular regions, with area A, base b, and height h.
   $b = 4.2$ cm, $h = 5.6$ cm, $A =$ ________
   $b = 18$ cm, $A = 63\ cm^2$, $h =$ ________
   $A = 48\ cm^2$, $h = 12$ cm, $b =$ ________

4. Complete, where d = number of distance units, r = rate of speed, and t = number of time units.
   $r = 45$ mi./hr., $t = 16$ hr., $d =$ ________
   $r = 55$ mi./hr., $d = 44$ mi., $t =$ ________
   $d = 300$ mi., $t = 0.75$ hr., $r =$ ________

5. At a warehouse, there are some boxes that hold 6 cubic feet. Two dimensions of the boxes are 1' 6" and 2'. How many boxes can be stacked on top of each other if the warehouse ceiling is 12' 3" high?

6. Last month the credit-card company added on $4.87 in interest to your uncle's bill. How much did your uncle owe, not counting the interest? The interest rate is 1.5% per month.

7. A worker at a park has to buy canvas to put around tennis court fences. The canvas is 6 feet wide. The worker measures and finds out he needs a piece 300 feet long. The canvas costs $1.89 per square foot. How much will the park worker have to pay for the canvas, before taxes?

8. A jet plane might go 600 miles per hour. How many seconds will it take the jet to go 5 miles?

9. A bag of fertilizer can cover 500 square feet. If the median of a highway is 32 feet wide, what length of median can a bag of fertilizer cover? How many bags are needed to cover one mile of median?

10. An engineer had 2.4 cm^3 of expensive metal to make a rod shaped like a rectangular solid. The rod had to have a cross-section 0.4 cm by 0.8 cm. What length of rod could she get from the metal?

A WORLDLY TRIP

FOCUS: Situational Activity

PURPOSE: The student will...

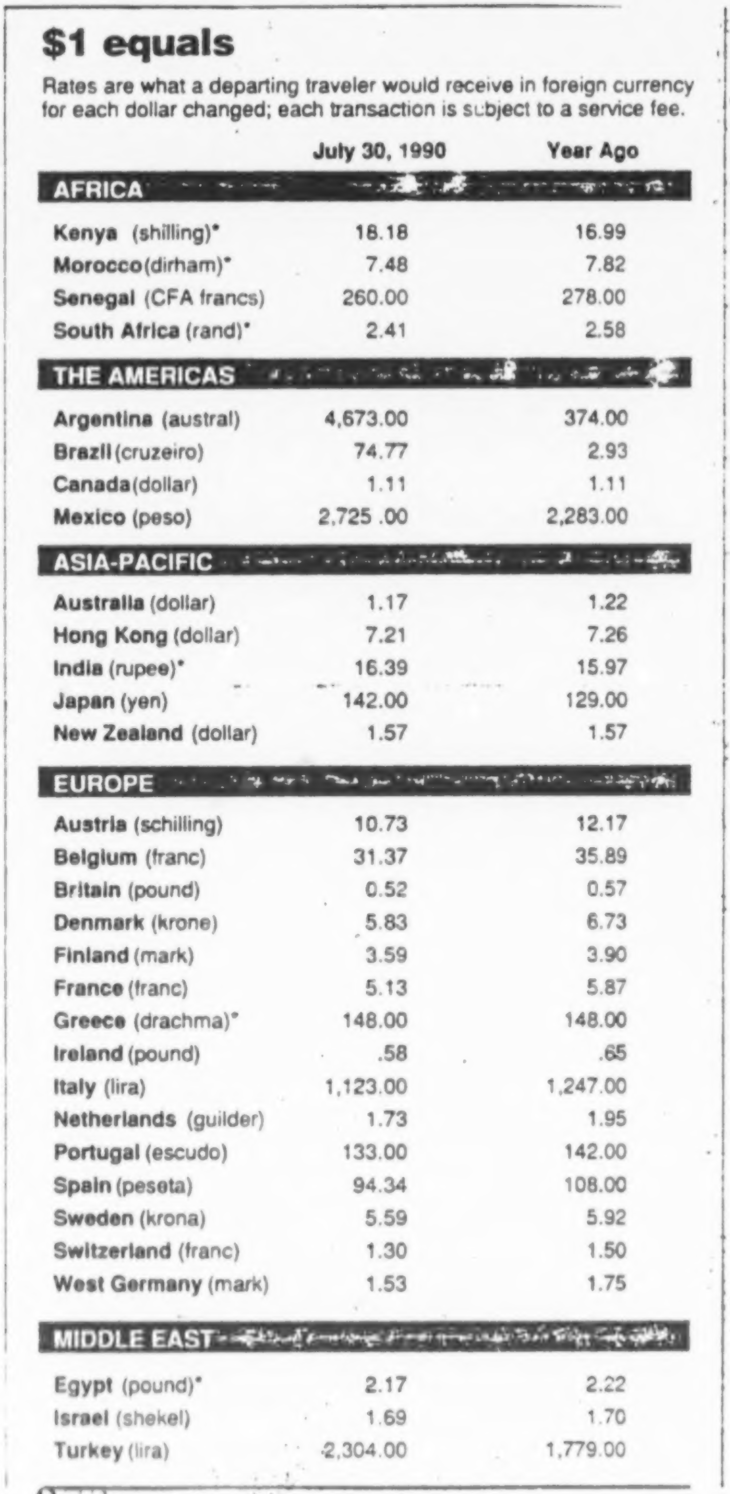
- Learn to work cooperatively on an assigned project;
- Research and write a brief report;
- Exchange U.S. currency to foreign currency; and
- Present the project to the class.

STUDENT BACKGROUND: Students need to be familiar with rate problems.

LESSON DEVELOPMENT: This project should be done with cooperative groups of four. They will be planning travel together as a group. Students will need to research the cost of a hotel, meals, and rental cars (optional) and obtain information from travel guides, travel agencies and other resources.

A WORLDLY TRIP

Plan a week (7 days, 6 nights) vacation to either Europe, Asia or any other geographic area below; plan to visit three countries. Transportation to and between countries has been paid for in advance as part of a package promotion. Plan a budget and account for hotel, meals, and extra expenses for transportation (while in any one country) and shopping for each country. Figure how much foreign currency you will need for each foreign country. Justify your expenditures day by day. Do some research and write a brief report about the attractions and/or activities you will be interested in for each country. Make up an itinerary of your day-to-day plans. The final report will be presented to the class. Visual aids can be used.



$1 equals

Rates are what a departing traveler would receive in foreign currency for each dollar changed; each transaction is subject to a service fee.

	July 30, 1990	Year Ago
AFRICA		
Kenya (shilling)*	18.18	16.99
Morocco (dirham)*	7.48	7.82
Senegal (CFA francs)	260.00	278.00
South Africa (rand)*	2.41	2.58
THE AMERICAS		
Argentina (austral)	4,673.00	374.00
Brazil (cruzeiro)	74.77	2.93
Canada (dollar)	1.11	1.11
Mexico (peso)	2,725 .00	2,283.00
ASIA-PACIFIC		
Australia (dollar)	1.17	1.22
Hong Kong (dollar)	7.21	7.26
India (rupee)*	16.39	15.97
Japan (yen)	142.00	129.00
New Zealand (dollar)	1.57	1.57
EUROPE		
Austria (schilling)	10.73	12.17
Belgium (franc)	31.37	35.89
Britain (pound)	0.52	0.57
Denmark (krone)	5.83	6.73
Finland (mark)	3.59	3.90
France (franc)	5.13	5.87
Greece (drachma)*	148.00	148.00
Ireland (pound)	.58	.65
Italy (lira)	1,123.00	1,247.00
Netherlands (guilder)	1.73	1.95
Portugal (escudo)	133.00	142.00
Spain (peseta)	94.34	108.00
Sweden (krona)	5.59	5.92
Switzerland (franc)	1.30	1.50
West Germany (mark)	1.53	1.75
MIDDLE EAST		
Egypt (pound)*	2.17	2.22
Israel (shekel)	1.69	1.70
Turkey (lira)	2,304.00	1,779.00

BEST-BUY THIRST QUENCHER

FOCUS: Situational Activity

PURPOSE: The students will...

- Work collaboratively to determine better buy, and
- Experience decision making related to a real-life situation.

STUDENT BACKGROUND: Students should be competent in dealing with rate problems, measurement, and graphing.

MATERIALS: BEST-BUY THIRST QUENCHER worksheet

LESSON DEVELOPMENT: The goal of the lesson is to determine which fast food restaurant gives you the most for your money with their soft drinks. Have the class list all the fast food restaurants in your area. Break the class into groups and assign each group a restaurant. The class then follows the directions on the worksheet. Determine a reasonable amount of time for the class to gather their data, analyze, and write up results for their presentations. You may want to lead the class through a brainstorming session to give them general ideas on how to get started. Review as necessary skills relating to rate, unit cost, measurement, and graphing.

After all groups have made their presentations and displayed their work in the classroom, have the class as a whole determine which restaurant has the best buy for each size of drink and have the class determine which group used the most efficient method of figuring the better buy. The class may notice if the same size of drink is the best buy at each of the different restaurants. Lead the class in a discussion about what might be the different factors to consider when a restaurant establishes the size and cost of its drinks.

SOURCE: Adapted from Junior High Math Lab, Action Math Associates, 1975

BEST-BUY THIRST QUENCHER

Your investigation group has just been assigned a fast-food restaurant. Your mission is to determine which size of soft drink gives you the most for your money. You will need to go to your restaurant, obtain a glass of each different size, and determine its unit cost. Use only regular prices, no special prices, unless you also include a seperate report reflecting any special prices offered. Make a chart and/or graph showing all sizes offered, their prices, each unit price, and any other pertinent information. Also, write up your conclusions explaining the procedure you used. Your write-up and visuals must be presented orally to the class and then displayed in the classroom.

After each group has made its presentation , the class as a whole will determine which restaurant has the best buy for a soft drink.

DON'T USE YOUR BRAIN

PURPOSE: The student will be alert to the necessity of thinking about the situation in a story problem.

TEACHER BACKGROUND: Students often respond mindlessly to story problems, perhaps even just picking out the numbers and trying something with them. (This procedure may even give correct answers to school story problems, especially ones involving whole numbers!) The point of occasionally using a problem like the following is to remind the students that they should think when solving your story problems.

SUGGESTED USE: Write the following on the board before class:

CAN YOU SOLVE THIS? A shepherd had 125 sheep and 5 goats. How old was the shepherd?

(This problem was used with European elementary school children, with about 75% giving an answer!) Subsequent discussion should touch on the fact that it is silly to try to solve school story problems without even thinking about whether they make sense.

Following are other problems of this sort, for occasional use. (It would probably not be fair to include such a problem on a test.) In such problems, you might ask your students what sensible questions could be asked, or what additional information would be needed to answer the question.

1. Some tapes were on sale for $2.95 each. Some CDs were on sale for $5.98. How much money does the store make in a 6-day week?

2. By 10:00 there were 125 cars in a parking lot. By 12:00, 43 more cars had arrived. Then at lunch 62 cars left, but 57 came back after lunch. At quitting time 118 cars left. What time was quitting time?

3. The rectangular space-station solar collector was 6.2 meters by 4.6 meters. Each square meter could make enough electricity to run the computer for an hour. How much did the solar collector cost?

FAST FOOD

FOCUS: Situational Lesson

- Equal groups
- Part of a whole
- Percent

PURPOSE: The students will . . .

- Analyze a favorite meal using charts to obtain nutritional information and convert data to grams/mg or percent as appropriate;
- Coordinate information in a fast food nutrition information table, RDA table, and menu to choose a healthful meal;
- Solve multi-step problems using all four operations with decimals;
- Write about mathematics in varied situations, making comparisons, justifying answers, and explaining reasoning;
- Use percents-of-a-whole in an application situation; and
- Make measurement conversions within the metric system and between the metric and English systems.

STUDENT BACKGROUND: Decimal number sense will be needed in estimating the amounts of varied items needed to meet nutrition requirements without exceeding the allowed fat, sodium, sugar, and Calorie figures. Students need to be able to find a percent of a number and to have percent sense as well. They will need to convert between grams and milligrams.

MATERIALS: Nutrition information from fast food restaurants (This is available at some restaurants upon request. Students may be asked to get the information in advance. If no such information is available, the accompanying charts from Jack in the Box and Carl's Junior can be used); menus or menu information for local fast food restaurants, and calculators.

Part 1: FAST FOOD, HEALTHFUL? worksheet
Part 2: A MORE HEALTHFUL MEAL worksheet

LESSON DEVELOPMENT:
Students who share the same taste in a "favorite fast food meal" might form groups to work on the problems in Part 1, then work independently in Part 2 (or continue in groups) to find a fast food meal that meets dietary guidelines. The questions on Fats (finding the percent fat in the favorite meal and in determining when it is appropriate to add percents and why) might be extension questions, or else they should be a teacher-led exercise. Groups might share solutions through group reports to be discussed by the class.

Students will have to find percents or amounts in grams/milligrams according to the way the information is presented by the restaurant and by the nutrition guidelines. Encourage use of mental computation and computational estimation throughout, with calculators used for exact answers only when they cannot be easily calculated mentally. Discuss the way to distinguish between sugars (e.g., in milkshakes or sweets) and complex carbohydrates (e.g., in whole wheat rolls and vegetables). Discuss the various ways *compare* is used in this lesson. Some situations are additive, some multiplicative, and some general usage.

EXTENSIONS: If you have students who understand spreadsheets, they could write one to use with this lesson.

ANSWERS:

FAST FOOD, NUTRITIOUS?

Answers vary. Example solution

Food	Per Serving		Fat	Sodium	Cholesterol	Carbohydrates
	Calories	Grams	grams	mg	mg	grams
Srdgh burger	712	223	50	1140	109	34
Side Salad	51	111	3	84	*	*
Bleu Cheese	262	70	22	918	18	14
Onion Rings	382	108	23	407+?	27	39
Choc Shake	330	322	7	270	25	55
Total	1410	524	98	2819	179	142

* Less than 1

Food	Percentage of U. S. Recommended Daily Allowance					
	Protein	Iron	Calcium	B1	B2	Niacin
Srdgh burger	50	24	19	43	28	40
Side Salad	7	**	6	4	6	**
Bleu Cheese	**	**	**	**	**	**
Onion Rings	8	8	3	14	7	9
Choc Shake	25	2	35	10	25	2
Total	90	34	63	71	66	51

** Less than 2% of RDA

Calories: $\dfrac{\text{Weight in pounds}}{2.2\ \dfrac{\text{lbs.}}{\text{kg}}}$ x (# from table) $\dfrac{\text{Calories per day}}{\text{kg}}$ = Calories per day

Fats:

To find the percent fat:

Grams of fat x $\dfrac{\text{9 Calories}}{\text{Gram of fat}}$ = Number of Calories from fats (e. g., 98 x 9 = 882)

Calories from fats ÷ Total Calories = Percent fat (e. g., 882 ÷ 1410 = 62.5%)

No, because the whole we are taking the percent of is not the same for each item since each has a different number of Calories

Finding the percent of the weight that is from fats is not the same as finding the percent of Calories from the fat (e.g., 98 grams fat ÷ 524 grams total = 18.7% ≠ 62.5%).

Fat contributes twice as many Calories to the diet per gram as do carbohydrates, so the percent of fat by weight is not the same as the percent of the Calories.

We can add the percents of protein each item contributes because each is a percent of the same thing (the RDA for protein in the diet), so the sum of the percents is the percent the sum is of the RDA.

# of grams of fat should have per day = 0.25 x # of Calories per day ÷ 9 Calories per gram of fat

Part of favorite meal = # of grams of fat in favorite meal ÷ # of grams of fat should have per day

The number of calories in my favorite meal, the amounts of fat, and the amount of sodium are all excessive.

A MORE HEALTHFUL MEAL

Answers vary.

Reference Information

Wallechinsky, D. & Wallace, I. (1981). *The people's almanac # 3.* New York: William Morrow.

FAST FOOD, NUTRITIOUS?

What is your favorite fast food meal? In the table below, list the items you would order for one typical complete meal.

The fast food restaurants have published nutrition information about the food they serve. Use this information to complete the tables below with the nutritional value of the meal you described.

My Favorite Meal

Food	Per Serving		Fat grams	Sodium mg	Cholesterol mg	Carbohydrates grams
	Calories	Grams				
Total						

Food	Percentage of U. S. Recommended Daily Allowance					
	Protein	Iron	Calcium	B1	B2	Niacin
Total						

Calories

Use the Calorie Table to find the number of Calories per kg per day appropriate for your age and gender. Use the information that 1 kg = 2.2 lbs. to find the number of Calories appropriate for you each day.

Calories Per Kilogram of Body Weight

Age (years)	Child	Female	Male
7-9	120		
10-12	100		
13-15		53	63
16-19		44	57
25		40	46
45		38	43
65		31	36

Record the number of Calories appropriate for you each day in the table below. How did you find it?

Dietary Guidelines

Calories	Protein	Fat	Sodium	Cholesterol	Carbohydrates (% of diet)	
					sugars	complex
________	12 %	25%	2 to 3 **grams**	300 mg	10 %	48 %

Fats

Dietary guidelines recommend that only 20% to 30% of your diet be made up of fats. To find the percent of your favorite meal that is fat, find the percent of the Calories that come from fat. Use the information that **one gram of fat is equivalent to 9 Calories**. Fats make up what percent of the Calories in your favorite meal? ______

How many grams of fat should you have each day? (Consider the number of Calories recommended for you. Assume 25% of your Calories should come from fat. Remember 1 gram of fat is equivalent to 9 Calories.) What part of this amount is in your favorite meal? ____________
Show your work.

Compare the information about your favorite meal with the Dietary Guidelines in the table. How healthful is your meal? What are the most serious problems with it? What effects might this meal have on your health if you have it often?

____

____

____

____

____

Extension:
If you find the percent of calories from fat in each item in your meal and add the percents, will you get the same answer for the percent of your meal that is fat? Why/why not? If you are not sure, estimate to decide. Explain how you reached an answer. ____________________________

____

Would the percent of the Calories coming from fats be the same as the percent of the weight (the grams) of the food that consists of fats?

______

How did you decide? ____

The nutrition information from some restaurants omits the percent of the daily recommended allowance of fat each item contains. Why do you think this might be? They give the percent for protein, so why not for fat? ____

____

Can we add the percents of protein each item contributes to our daily requirements (RDA) to find the percent of the RDA of protein the meal provides? ____ Why or why not?______________________________

____

How is this different from adding the percents of fat in the question above? ____

____

A MORE HEALTHFUL MEAL

If having your favorite fast food meal often seems likely to lead to health problems, can you choose a more healthful meal from a fast food restaurant? Try to choose a meal that meets **one half to one third** of a day's requirements in each nutrition category. Use the information in Fast Food for guidelines. Be careful to avoid excess fats, sodium, sugars, and calories.

My Healthful Meal

Food	Per Serving		Fat grams	Sodium mg	Cholesterol mg	Carbohydrates grams
	Calories	Grams				
Total						

Food	Percentage of U. S. Recommended Daily Allowance					
	Protein	Iron	Calcium	B1	B2	Niacin
Total						

Describe how you decided on a meal that meets all of the guidelines.

Compare the cost of this nutritious meal with the cost of your favorite meal. Show the equation you would use.

The Average American

The "average American" consumes 60 times as much salt as is needed. How about you? List the foods you would eat for one typical day. By finding items similar to those on your list (in the restaurant nutrition information you used to get the information about your favorite meal), use the amounts of sodium in the various fast food items to estimate the amount of sodium you consume each day. Do you eat more than is recommended? If so, how many times as much?
Show how you determined your answer.

In 1980, almost 9 of every 10 Americans ate at a fast food place at least once a month. The "average fast food customer" ate fast food 8 times a month. Do you think the same would be true today? Compare these figures with the habits of your family.

In 1978, the "average American" spent $85.62 at fast food restaurants. Estimate how this compares with what you will spend this year. Show your method of estimating.

Do you think the difference is due to inflation or to a difference in the number of times you eat fast food compared to the "average American" in 1978? ____

____

When eating out, the "average American" under age 35 will pick a fast food restaurant 3 times out of 5. How does that compare with your experience? What fraction of the times that you eat out do you have fast food? Is that more or less often than this "average American?"

____

____

Quarter-Pounders

A spider eats its own weight in food every day. How many quarter-pounders would you need to eat in one day to match this feat for your weight? __________ (Assume a quarter-pounder weighs a quarter pound.)
Give two ways you could reach the answer.

(Assume a quarter-pounder has 584 Calories unless your restaurant information has the number.)
About how many days' calories would that be for you? ____________
Explain the method you used to reach this answer.

____

Each day, we actually consume food weighing the equivalent of 1% to 2% of our body weight. **Estimate** the number of quarter-pounders this would be for your weight. ______________

If you did consume 1.5% of your weight in quarter-pounders each day, how many calories would that be each day? ________
Show your work.

If you really eat this many quarter-pounders each day, will you gain or lose weight? _____ Explain.

What other problems would this cause? ______________________

____

Hamburgers/Sandwiches	Serving size grams	Calories per serving	Protein grams	Carbohydrate grams	Fat grams	Saturated fatty acids grams	Polyunsaturated fatty acids grams	Cholesterol mg/serving	Sodium mg/serving	Potassium mg/serving	Percentage of U.S. Recommended Daily Allowance							
											Protein	Vitamin A	Vitamin C	Thiamin B1	Riboflavin B2	Niacin	Calcium	Iron
Famous Star Hamburger™	231	590	24	42	36	13	8	45	890	350	35	10	6	20	30	40	15	35
Super Star® Hamburger	301	770	37	44	50	21	5	125	990	590	50	15	8	25	30	60	15	40
Western Bacon Cheeseburger®	213	640	33	50	34	14	3	108	1375	393	51	8	4	37	28	31	25	30
Double Western Bacon Cheeseburger™	294	900	42	62	54	24	4	148	1580	568	61	10	4	77	33	46	35	45
Old Time Star® Hamburger	168	400	24	38	17	7	1	80	760	345	35	6	6	20	30	40	15	25
Hamburger	86	220	12	26	8	4	0	45	445	145	20	2	2	15	10	25	10	10
Lite Menu Charbroiler BBQ Chicken Sandwich™	178	320	28	40	5	2	2	50	955	400	45	4	2	30	10	30	8	20
Charbroiler Chicken Club Sandwich™	234	520	26	54	23	1	2	88	1125	413	41	4	4	47	23	41	25	25
California Roast Beef 'n Swiss™	209	360	31	43	8	4	1	130	1070	375	50	20	4	35	20	30	35	30
Filet of Fish Sandwich	223	550	22	58	26	11	5	90	945	375	30	*	4	35	15	20	25	20
American Cheese	18	63	4	1	5	3	†	16	290	13	9	4	*	*	4	*	13	*
Swiss Cheese	18	57	4	1	4	3	0	16	221	16	9	3	*	*	3	*	16	*
"Great Stuff"™ Potatoes																		
Fiesta Potato	432	550	25	60	23	9	7	40	1230	1285	35	15	*	20	25	25	30	40
Broccoli & Cheese Potato	398	470	15	61	17	5	7	10	690	1075	20	25	30	15	8	30	25	50
Bacon & Cheese Potato	400	650	23	63	34	12	12	45	1820	1125	35	8	*	20	15	20	30	45
Sour Cream & Chive Potato	294	350	8	49	13	5	4	10	140	960	10	*	4	10	8	15	8	30
Cheese Potato	403	550	18	72	22	7	8	40	785	1200	25	10	6	20	10	20	35	30
Lite Menu Lite Potato	278	250	8	54	3	0	0	0	35	920	10	*	6	6	4	15	8	20
Breakfast																		
Sunrise Sandwich® w/Bacon	123	370	16	32	19	6	4	119	643	147	24	*	*	17	17	11	15	20
Sunrise Sandwich® w/Sausage	170	490	21	31	31	11	5	161	923	209	33	5	*	15	19	20	15	20
French Toast Dips***	132	480	8	54	25	10	4	54	576	114	12	5	*	12	12	12	10	18
Scrambled Eggs	67	120	9	2	9	4	1	245	105	90	15	25	*	4	10	*	4	10
Hot Cakes w/Margarine**	156	360	7	59	12	3	3	15	1190	200	10	*	*	15	20	25	4	25
English Muffin w/Margarine	57	180	4	28	6	2	9	0	275	60	6	*	*	6	8	4	8	15
Sausage – 1 patty	44	190	7	1	17	4	1	25	275	70	16	*	*	6	4	10	*	6
Bacon – 2 strips	10	60	3	1	5	2	1	11	160	48	7	*	*	7	*	3	*	*
Hash Brown Nuggets	85	170	2	20	9	4	5	0	350	350	4	*	6	6	*	8	*	4

*Less than 2% in all cases **Syrup not included ***Labels will vary depending on condiments †Trace

¹The information on cholesterol content is provided for individuals who on the advice of a physician are modifying their dietary intake of cholesterol. Due to fluctuations in food suppliers, nutritional figures may vary slightly.

U.S. Recommended Daily Allowance (USRDA): a guideline established by the Food & Nutrition Board (National Research Council of National Academy of Sciences) estimating daily required nutrient intake based on people with the highest needs. Therefore, the percentages tell you how much of the USRDA a particular food supplies.

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	Serving size grams	Calories per serving	Protein grams	Carbohydrate grams	Fat grams	Saturated fatty acids grams	Polyunsaturated fatty acids grams	Cholesterol mg/serving	Sodium mg/serving	Potassium mg/serving	Percentage of U.S. Recommended Daily Allowance							
											Protein	Vitamin A	Vitamin C	Thiamin B1	Riboflavin B2	Niacin	Calcium	Iron
Bakery Products																		
Blueberry Muffin	120	340	5	61	9	1	4	36	300	188	11	4	•	12	12	12	3	18
Bran Muffin	113	258	5	44	6	0	5	50	310	19	5	•	•	9	8	12	•	12
Danish (varieties)	99	300	8	49	9	3	5	0	550	77	11	•	•	23	15	15	3	15
Chocolate Chip Cookie 2.5 oz.	70	353	3	45	16	7	2	15	202	110	6	4	•	4	6	10	4	10
Fudge Brownie	92	430	6	64	19	5	14	†	210	117	9	•	•	7	6	4	•	10
Side Orders																		
French Fries (regular size)	126	420	4	54	20	5	0	0	195	675	6	0	10	15	2	15	0	6
Zucchini	163	380	8	44	19	4	0	0	1120	395	10	10	10	15	15	10	4	15
Onion Rings	151	520	9	63	26	6	1	0	960	155	10	0	4	15	10	15	4	20
Entree Salads-To-Go™																		
Lite Menu Small Garden Salad-To-Go	117	46	2	4	2	1	0	7	57	226	4	20	30	8	8	2	6	6
Lite Menu Large Garden Salad-To-Go	240	95	5	9	5	2	0	14	116	473	12	49	60	13	14	2	13	13
Lite Menu Chicken Salad-To-Go	309	206	23	12	8	3	0	83	453	777	35	50	60	15	20	25	15	20
Taco Salad-To-Go	406	356	29	18	19	6	0	99	690	937	45	80	60	20	40	15	25	30
Salad Dressings***																		
Italian Dressing – 1 oz.	28	120	0	1	13	2	6	0	210	20	•	•	•	•	•	•	•	•
House Dressing – 1 oz.	28	110	1	2	11	3	3	10	170	35	•	•	•	•	2	•	2	•
Blue Cheese Dressing – 1 oz.	28	151	1	0	15	3	4	18	255	22	•	•	•	•	•	•	•	•
1000 Island Dressing – 1 oz.	28	110	0	4	11	3	5	5	200	55	•	2	•	•	•	•	•	2
Reduced Calorie French Dressing – 1 oz.	28	38	0	5	2	0	2	0	292	5	•	•	•	•	•	•	•	•
Beverages/Shakes																		
Carbonated Beverages – Reg. size	600	243	0	62	0	0	0	0	37	0	•	•	•	•	•	•	•	•
Diet Carbonated Beverage – Reg. size	600	2	0	0	0	0	0	0	13	63	•	•	•	•	•	•	•	•
Iced Tea – Reg. size	600	2	0	0	0	0	0	0	0	0	•	•	•	•	•	•	•	•
Milk 2% lowfat – 10 fl. oz.	311	175	13	16	6	3	2	0	181	550	20	5	4	8	38	•	44	•
Orange Juice – Small size	249	[illegible]	2	21	1	†	†	0	2	404	3	3	117	16	3	3	2	2
Shakes – Reg. size	330	353	11	61	7	4	0	17	255	395	15	•	•	10	35	2	43	10
Other Specialities																		
Jr. Crisp Burrito™ (each)	43	140	5	13	7	3	1	65	200	70	8	6	•	4	8	4	4	6
Taco Sauce	29	8	0	2	0	0	0	0	160	65	0	•	•	•	•	•	•	•
Guacamole	28	55	1	4	4	1	1	0	142	180	•	2	6	•	•	•	4	•
Salsa	29	8	0	2	0	0	0	0	210	75	•	50	•	•	•	•	•	•

Rev. [illegible]

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SUMMARY OF NUTRITIONAL VALUES

										Percent of U.S. Recommended Daily Allowance							
Menu Item	Serving Size (grams)	Calories (per serving)	Protein (grams)	Carbohydrates (grams)	Fat (grams)	Saturated Fatty Acids (grams)	Monounsaturated Fatty Acids (gms)	Polyunsaturated Fatty Acids (gms)	Cholesterol (milligrams)	Sodium (milligrams)	Protein	Vitamin A	Vitamin C	Thiamin (B_1)	Riboflavin (B_2)	Niacin	Calcium	Iron
Egg Rolls – 3 Piece	171	405	15	42	19	7.2	1.8	8.1	30	903	20	10	8	15	10	15	5	15
Egg Rolls – 5 Piece	285	675	26	70	32	12.0	3.0	13.5	50	1505	35	15	15	25	15	25	8	25
Chicken Strips – 4 Piece	125	349	29	28	14	6.8	5.9	0.7	68	748	40	10	*	4	8	10	10	8
Chicken Strips – 6 Piece	187	523	43	42	20	10.0	8.9	1.0	103	1122	60	20	*	8	10	15	20	10
Shrimp – 10 Piece	84	270	10	22	16	7.2	6.5	1.9	84	669	10	15	*	2	6	6	20	6
Shrimp – 15 Piece	125	404	15	34	24	10.8	9.8	2.9	126	1003	20	20	*	4	8	8	35	8
Taquitos – 5 Piece	140	363	16	[illegible]	16	5.6	6.6	2.4	37	467	24	*	*	4	8	11	17	18
Taquitos – 7 Piece	195	508	22	56	22	7.9	7.9	3.4	52	654	33	*	*	5	11	15	23	25
Sweet & Sour Sauce	28	40	**	11	**	**	**	**	**	160	*	*	*	*	*	*	*	*
BBQ Sauce	28	44	0.5	10.6	**	**	**	**	0	300	*	*	*	*	*	*	*	*
Seafood Cocktail Sauce	28	32	**	6.8	**	**	**	**	0	206	*	*	*	*	*	*	*	2
Guacamole	25	55	0.9	1.8	5	—	—	—	0	130	*	*	*	*	*	*	2	*
Salsa	25	8	0.2	2.0	**	**	**	**	0	129	*	*	*	*	*	*	*	*
Taco	81	191	8	16	11	5.2	4.4	**	21	406	12	8	*	5	10	5	10	8
Super Taco	135	288	12	21	17	8	6.9	1.2	37	765	19	12	3	8	5	7	15	9
Small French Fries	68	221	2	27	12	5	4.9	1.7	8	164	4	*	5	5	2	6	*	3
Regular French Fries	109	353	3	43	19	7.9	7.8	2.7	13	262	6	*	8	8	3	9	*	5
Jumbo Fries	136	442	4	54	24	10	9.8	3.4	16	328	8	*	10	10	4	12	*	6
Onion Rings	108	382	5	39	23	11.1	9.3	1.3	27	407	8	*	5	14	7	9	3	8
Sesame Breadsticks	16	70	2	12	2	—	—	—	**	110	*	*	*	*	*	*	*	*
Tortilla Chips	28	139	2	18	6	—	—	—	**	134	*	*	*	*	*	*	*	*
Hot Apple Turnover	119	410	4	45	24	10.8	10.3	2	15	350	6	*	*	15	8	10	*	8
Cheesecake	99	309	8	29	17.5	9	7	1	63	208	12	*	*	3	14	10	11	3
Orange Juice	183	80	1	20	0	0	0	0	0	0	2	8	160	10	4	4	2	2
Lowfat Milk	244	122	8	12	5	2.9	**	1.2	18	122	20	10	4	6	25	*	30	*
Vanilla Milk Shake	317	320	10	57	6	3.6	1.8	**	25	230	20	*	*	10	20	2	35	*
Chocolate Milk Shake	322	330	11	55	7	4.3	2.1	**	25	270	25	*	*	10	35	2	35	4
Strawberry Milk Shake	328	320	10	55	7	4.3	2	**	25	240	25	*	*	10	25	2	35	2
Coca-Cola classic®	12°	144	0	36	0	0	0	0	0	14	*	*	*	*	*	*	*	*
Diet Coke®	12°	0.8	0	**	0	0	0	0	0	26	*	*	*	*	*	*	*	*
Ramblin' Root Beer®	12°	176	0	46	0	0	0	0	0	20	*	*	*	*	*	*	*	*
Sprite®	12°	144	0	36	0	0	0	0	0	46	*	*	*	*	*	*	*	*
Dr. Pepper®	12°	144	0	37	0	0	0	0	0	18	*	*	*	*	*	*	*	*
Iced Tea	12°	3	0.8	**	0	0	0	0	0	4.5	*	*	*	*	*	*	*	*
Coffee	8°	2	0	**	0	0	0	0	0	26	*	*	*	*	*	*	*	*

*Contains less than 2% of the U.S. Recommended Daily Allowance **Less than 1 †Excluding sauce °Fluid ounces

Menu Item	Serving Size (grams)	Calories (per serving)	Protein (grams)	Carbohydrates (grams)	Fat (grams)	Saturated Fatty Acids (grams)	Monounsaturated Fatty Acids (gms)	Polyunsaturated Fatty Acids (gms)	Cholesterol (milligrams)	Sodium (milligrams)	Percent of U.S. Recommended Daily Allowance							
											Protein	Vitamin A	Vitamin C	Thiamin (B_1)	Riboflavin (B_2)	Niacin	Calcium	Iron
Scrambled Egg Pocket	183	431	29	31	21	7.5	7.4	2.3	354	1060	45	21	*	45	40	23	21	20
Supreme Crescent	146	547	20	27	40	13.2	18.9	7.8	178	1053	31	11	*	43	32	21	15	15
Sausage Crescent	156	584	22	28	43	15.5	21.5	5.7	187	1012	34	11	*	40	30	23	17	16
BREAKFAST JACK®	126	307	18	30	13	5.2	5	2.5	203	871	29	9	*	31	24	15	17	17
Scrambled Egg Platter	249	662	24	52	40	17.1	16.0	4.7	354	1188	40	20	8	20	45	25	20	30
Hash Browns	62	116	2	11	7	3.6	3.2	0.4	3	211	2	*	*	4	2	6	4	2
Pancake Platter	231	612	15	87	22	8.6	7.6	3.5	99	888	25	6	10	2	50	35	10	10
Pancake Syrup	42	181	0	30	0	0	0	0	0	6	*	*	*	*	*	*	*	*
Grape Jelly	14	38	0	9	0	0	0	0	0	3	*	*	*	*	*	*	*	*
Hamburger	96	267	13	28	11	4.1	4.9	2.0	26	556	15	*	*	10	15	10	15	10
Cheeseburger	112	315	15	33	14	5.7	5.9	2.3	41	746	20	4	*	15	15	15	25	15
Double Cheeseburger	149	467	21	33	27	12.3	11.6	3.1	72	842	30	8	*	10	20	30	40	15
JUMBO JACK®	222	584	26	42	34	11	13	8	73	733	40	*	*	24	17	9	14	17
JUMBO JACK® with Cheese	242	677	32	46	40	14	15	9	102	1090	49	*	*	24	26	8	27	21
Bacon Cheeseburger	230	705	35	48	39	15	15	7	85	1127	54	*	*	32	31	15	28	22
Grilled Sourdough Burger	223	712	32	34	50	15.9	17.8	7.9	109	1140	50	14	*	43	28	40	19	24
Swiss and Bacon Burger	187	678	31	34	47	20	18	7	92	1458	48	*	*	17	20	17	22	12
Ultimate Cheeseburger	280	942	47	33	69	26.4	24.2	18.1	127	1176	70	15	*	20	30	40	60	35
Beef Fajita Pita	175	333	24	27	14	5.9	5.3	2.2	45	635	35	10	*	25	30	25	25	60
Chicken Fajita Pita	189	292	24	29	8	2.9	3.6	1.4	34	703	35	10	*	50	10	30	25	15
Grilled Chicken Fillet	205	408	31	33	17	4.1	4.6	6.0	64	1130	48	4	13	18	22	72	17	12
Chicken Supreme	231	575	27	34	36	14.3	13.4	7.6	62	1525	40	*	*	20	8	60	10	20
Fish Supreme	228	554	20	47	32	13.5	11.0	7.4	66	1047	30	30	*	10	15	25	45	15
Chef Salad	332	325	30	10	18	8.4	5.2	1.3	142	900	46	73	46	29	27	30	44	8
Taco Salad	402	503	34	28	31	13.4	11.9	1.6	92	1600	53	27	15	19	31	29	41	21
Mexican Chicken Salad	413	442	28	30	23	8.6	7.9	2.8	89	1500	43	25	30	17	27	30	39	11
Side Salad	111	51	7	**	3	2	1	0	**	84	11	*	*	4	6	*	6	*
Buttermilk House Dressing	70	362	**	8	36	5.8	8.4	21.6	21	694	*	*	*	*	*	*	*	*
Bleu Cheese Dressing	70	262	**	14	22	4	5.4	12.6	18	918	*	*	*	*	*	*	*	*
Thousand Island Dressing	70	312	**	12	30	5	7.2	17.6	23	700	*	*	*	*	*	*	*	*
Reduced-Calorie French Dressing	70	176	**	26	8	1.2	1.8	4.8	0	600	*	*	*	*	*	*	*	*

*Contains less than 2% of the U.S. Recommended Daily Allowance **Less than 1 †Excluding sauce ‡Fluid ounces

GLOBAL CONSERVATION

FOCUS: Situational Activity

PURPOSE: The student will...

- Use resources in problem solving strategies;
- Read to solve;
- Write to solve and communicate;
- Give oral presentation; and
- Work as an integral part of a team effort.

STUDENT BACKGROUND: Students will have to know how to use their library to gather data for research and reporting. They should be comfortable reading and creating tables, charts, and graphs. It is preferred that they also know how to use a word processor to write their essay.

Students need to have a firm understanding of the fundamental operations that will be necessary to comprehend gathered data and perform statistical research.

TEACHER BACKGROUND: It is important to use mathematics in a meaningful way. Students should see that math skills can be combined with other academic abilities to solve problems in a natural setting. It is also necessary to learn that collaboration with others can be very effective when solving difficult problems, especially in terms of multiple perspectives and varied approaches.

MATERIALS: GLOBAL CONSERVATION activity sheet, current newspapers, periodicals, and a calculator.

LESSON DEVELOPMENT: For this situational project to be successful, students must leave the class room to gather data. They should be exposed to as much exploratory opportunity as possible. This means that provisions should be made in advance for students to have access to all your school's resources, library, media, modems, and computer labs.

It is essential for the teacher to be concerned with the student as a learner, researcher, and reporter, not just a computer of mathematical operations.

Before this activity is assigned it will require a discussion about the state of the earth. It is suggested that the students be given relevant reading materials, preferably current-event type to study the night before. Then, in a Socratic seminar, facilitate a discussion in a manner that challenges and motivates your students to make this problem personal.

Although the question has been pre-written, the teacher should take the option of rewriting it to address or accommodate any special or unique circumstances that might emerge out of the discussion. Letting the students rewrite it can also have reciprocal teaching value.

If this lesson is going to have any special problems or difficulties it will be primarily logistical. Scheduling time for writer's labs, library, classroom writing and group collaboration, computer labs, and final reporting is necessary for all-around success. It is important to be flexible, and to BE PREPARED!

ANSWER: Openended. It is important for students to recognize that the earth is a fragile planet and that it requires a balance in all things. It might also be significant to report on the role of human beings, especially since we are destroying so much of the balance.

GLOBAL CONSERVATION

The earth is facing more challenges than ever. We are losing forests at an alarming rate. Famine, drought, and hunger are killing us. Our population is expanding exponentially. Our protective ozone layer is being destroyed. Global economies struggle as unemployment, homelessness, and crime increase. New diseases emerge. Animal species become extinct. And the scenario continues...

You are the hope for the future! It is your task to come to terms with this dilemma. You and your team can choose to study/research any or all of earth's challenges. The important thing is for you to determine a general, global theme, potential solutions to the problem, and a sound conclusion. (What is the state of the earth? What is the primary problem? Is there a primary problem? Must we solve the problems individually? Can we solve them together? Are the problems connected? What is the role of the human being?, etc)

Type a three page essay to report your team's research into this problem. Then make arrangements for a ten minute presentation to the class. Make it interesting. Use charts, graphs, tables, slides or videos (that you produce) and any other visual aids that will be helpful.

NEWSWORTHY PROBLEMS

FOCUS: Application Activity

PURPOSE: The student will . . .

- Use a current publication as a source for information on which to base word problems;
- Solve the problems they write; and
- Solve the problems written by another student or group.

STUDENT BACKGROUND: Students should have some experience writing word problems and with solving problems having insufficient data and extraneous quantitative information.

MATERIALS: Current newspapers or weekly news magazines for each student or for each group; NEWSWORTHY PROBLEMS worksheet.

LESSON DEVELOPMENT: The teacher may model the process of locating an article with quantitative information, reading, and thinking aloud about possible quantitative word problems that could be written based on the information in the article. Students may be given further constraints on the problems they write according to the operation meanings studied and the language used in class to describe the operation situations. They may be asked, for example, to write one problem about a sharing equally division situation, one for repeated subtraction, etc.

It has been suggested in the instructions to the students that they write one problem with insufficient data. The problem solvers should be told to explain what other information is needed to solve the problem if they encounter this situation.

Remind students that they should provide an answer key for their problems. Each group may work the problems of another group and during class discussion of the problems, suggest the best ones to be polished for a problem set to be given to another similar class to work.

ANSWERS: Vary

NEWSWORTHY PROBLEMS

A. Using a current newspaper or weekly news magazine, write at least five word problems. Use information from an article in each of the following sections: world news, local news (if using a newspaper), sports, entertainment, and the economy.

B. Also, write two multi-step problems based on advertisements from anywhere in the publication.

In your problems, frequently include numerical information from the article not needed to solve your problem. You might want to omit some necessary information from **one** of your problems. For such problems, your classmates would be expected to explain what additional information would be needed to solve the problem. Be sure you can solve the problems and provide an answer key for them.

OPEN-ENDED PROBLEMS

FOCUS: Application Activity

PURPOSE: The student will ...

- Have experience with open-ended problems.

TEACHER BACKGROUND: A compelling case can be made that school mathematics often leads students into thinking that each problem has just one answer, perhaps obtainable in only one way. And, in many curricula students are rarely asked to make up problems. The use of open-ended problems is one means of showing students that problems may have many answers and indeed there may be many questions that could be asked for a given situation.

MATERIALS: OPEN-ENDED PROBLEMS worksheet, and similar problems.

LESSON DEVELOPMENT: Problems like the ones that follow could periodically serve as the focus for part of a period. With the problem displayed, students could think about the problem individually for 2-3 minutes, and then collectively in a small group, with a group report (written or oral) following. Highlight the variety of solutions or questions that the groups come up with.

Encouraging a competition among the groups for the greatest number of solutions or questions, perhaps as an on-going competition over two to three days, adds a little incentive for doing more than a minimum.

SOURCE: Idea (and problems 1-3) are from Edward Silver and Verna Adams, "Problem Solving: Tips for Teachers," Arithmetic Teacher, (May 1987), 34(9), pp. 34-35. See also Edward Silver and Margaret Smith, "Teaching Mathematics and Thinking," Arithmetic Teacher, (April 1990), 37(8), pp. 34-37.

OPEN-ENDED PROBLEMS

1. A rectangle has a perimeter of 36 centimeters. Its area is larger than 50 square centimeters. What conclusions about the rectangle can you draw?

2. Write and solve as many problems as you can, starting with this information:
   Each of Thu and Maria rides a bike to school. Thu lives 8 blocks from school. Maria lives 12 blocks from school. It takes Thu 16 minutes to ride his bike to school each morning.

3. You have been saving all your change, and you now have a pocketful. A can of soft drink from a machine costs $0.60. What coins should you use to get rid of as much change as possible.

4. Write and solve as many problems as you can, starting with this information:
   When you are 16, you can get a minimum-wage job that pays $4.25 per hour, before deductions.

5. Write and solve as many problems as you can, starting with this information:
   When you drop one kind of ball, it bounces back up 35% as high as from where it started.

6. Write and solve as many problems as you can, starting with this information:
   Tony Gwynn is batting .324. Will Clark is batting .306.

7. Write and solve as many problems as you can, starting with this information:
   Valentine Girl (New Kids on the Block) 3 min. 57 sec. (3:57)
   Dancing Machine (Michael Jackson) 3:10
   Coat of Many Colors (Dolly Parton) 2:53
   Lonesome Rodeo Cowboy (George Strait) 4:25
   Magic Flute Overture (Wolfgang Mozart) 6:38
   Oh, Bess, Oh Where's My Bess? (George Gershwin) 2:54
   You Really Got Me Now (Jon Bon Jovi) 2:23

PLAYGROUND

FOCUS: Situational Activity

PURPOSE: The students will...

- Design a scale drawing of a playground;
- Use visualization and estimating while creating their playground;
- Research possible layouts and costs for equipment;
- Work with other students in their decision making; and
- Give written and oral presentations of their playground.

STUDENT BACKGROUND: Students should have experience with scale drawings, rates, and measurement.

MATERIALS: PLAYGROUND worksheet, graph paper (large graph paper is nice for oral presentations but not necessary), colored pens or pencils (optional), and rulers.

LESSON DEVELOPMENT: Divide the class into small groups. Discuss with the class what their task is and brainstorm possible procedures for designing the playground, finding equipment, and determining costs for equipment and possible labor. Suggest that they assign a different responsibility to each group member. Set a time limit for creating their playground and presentation, at least two weeks. You may want to make this a contest and give out awards for best plan, most cost effective, safest, etc.

SOURCE: Idea from SPACES; Lawrence Hall of Science; 1982

PLAYGROUND

Your neighborhood elementary school would like to re-do their playground. Much of their equipment has been damaged or stolen. The school has decided to take everything out and start from scratch. Since they want to please everyone, they are gathering input from other schools, teachers, parents, and students. Because you have been a professional user of playgrounds for at least the past six years of your life, the school would like you to create a new design. They want you to create the ultimate playground to be used by first through sixth graders.

On the graph paper provided, you are to design a playground to scale. You will be making an oral presentation to the "elementary school" (your classmates and teacher), displaying your drawing, explaining all the wonderful features, and giving the approximate cost. You will also have to turn in all this information in writing, including an itemized list of equipment and their costs. Also include the source you used for determining the cost of each item.

All decisions are yours except for the dimensions of the playground. The field measures 200 feet long by 150 feet wide. You pick an appropriate scale to use on your drawing. You will also have to do some research to determine approximate costs of your equipment and possible landscaping. (Do not worry about balls, bats, nets, etc.; you are responsible for only things that are permanently on a playground). If you cannot find the cost of an item, make an educated guess. There is no budget, but keeping costs down is always appreciated. Make sure your final selection of equipment is realistic.

Here are some other considerations when designing your playground:

- Cost
- Spacing of equipment
- Labor
- Aesthetics
- Safety
- Grass
- Trees (There are already three trees that can be transplanted.)

Possible sources for determining costs:

- Yellow pages
- Catalogs
- Parks and Recreation Department
- Schools
- School District Office
- Construction Workers
- Home Improvement Stores

Possible student jobs:

- Draftsperson
- Engineer
- Finance manager
- Presenters

(Duties may be held by one or more people)

PUBLIC SMOKING

FOCUS: Situational Application Activity

PURPOSE: The student will...

- Use mathematics in a meaningful way;
- Formulate problems from within and without mathematics;
- Persue an open-ended problem;
- Reason with graphs, tables, and charts;
- Reason inductively and deductively;
- Connect mathematics to the world outside the classroom;
- Use statistical methods to describe, analyze, evaluate and make decisions;
- Read to solve;
- Use resources to solve and communicate;
- Write to solve and communicate;
- Give an oral presentation; and
- Work as an integral part of a team effort.

STUDENT BACKGROUND: Students will have to know how to use their library to gather data for research and reporting. They should be comfortable reading, understanding, and creating tables, charts, and graphs. It is preferred that they also know how to use a word processor.

TEACHER BACKGROUND: It is important to use mathematics in a meaningful way. Students should see that math skills can be combined with other academic abilities to solve problems in a natural setting. It is also necessary to learn that collaboration with others can be very effective when solving difficult problems, especially in terms of multiple perspectives and varied approaches.

MATERIALS: PUBLIC SMOKING activity sheet

LESSON DEVELOPMENT: This lesson should be assigned when students have a firm understanding of fundamental operations that will be necessary to comprehend gathered data and perform statistical research. This means that some time during the second semester would be best.

For this situational lesson to be successful, much of the work will have to be done outside the classroom. The teacher should either assign some current event type reading the night before or supply a copy of some clippings previously prepared. The purpose is for a seminar the next day.

The teacher's role in the seminar is to facilitate a discussion that will provoke ideas and perhaps develop spontaneous groups that are eager to work together for common goals. Be sure to watch for groups with divergent interests. They sometimes offer prime opportunities for debates.

After the discussion, pass out the assignments. Be clear that they may work as individuals or in groups. Try to encourage teamwork. Make sure they understand there is a paper to write and an oral presentation to give.

In order to be successful there should be some time scheduled for the library and the computer lab. Some writing time may also be necessary in class. This usually can be accomodated when students are finished with the day's assignment. Handled in this way, it can act as enrichment.

Students should be encouraged to write special interest groups to get information from primary sources. Guest speakers and field trips could also have special effect.

ANSWER: Will vary, but at least two concerns should be addressed, human rights, and health.

PUBLIC SMOKING

Make a case for/against smoking in public places. Be sure to research and discuss some pertinent issues. Human rights will play an important role. Look at smoking on airlines, in resturants, public bathrooms, airports, shopping centers/malls, super markets, theaters, civic and convention centers, and any enclosed area (public or private) that has massive public traffic. Second-hand smoke needs to be addressed, especially as it relates pregnant mothers.

Give special consideration to arguments made by the cigarette industry. Note any special connections, especially snuff, chewing tobacco and alcohol. Regard advertisements as viable research. What is the attraction to cigarettes?

Compare/contrast the cigarette argument with ones made by medical groups and associations, especially the Cancer Society, Lung Association, and the Surgeon General. Note any special connections, especially drug and alcohol.

Support your case with evidence, especially graphs, charts, tables, and professional/medical journals that relate to mortality and diseases atributed to smoking.

Finally, write a three to five page essay that resolves the issue between human rights and public law as it specifically relates to smoking in public places. Be prepared to defend it with a 5-10 minute oral presentation.

STUDENT SURVEY

FOCUS: Situational Activity

PURPOSE: The student will...

- Survey the student population at his/her school;
- Project how a larger population feels about the question(s) using the survey data collected at the school;
- Use graphs and/or other visual aids to represent the survey data; and
- Work collaboratively with a small group in writing, collecting, and presenting the survey to the class.

STUDENT BACKGROUND: Students will need to have proportion sense, and have had experience with graphing.

MATERIALS: STUDENT SURVEY worksheet

LESSON DEVELOPMENT: This activity is intended for cooperative groups of four. The students' first task is to come up with a list of questions they would like to use in their survey. It may be necessary for you to preview the questions and approve the appropriate ones (and at the same time make sure there are no duplicates). When conducting the survey, either in written form or orally, students are required to survey a cross section of ethnic students, a fair representation of the larger population. Discuss the percentage of each ethnic group present at your school (they may need to ask the principal or a school administrator what those percents are), the entire school district and community, and why it is important to survey all groups. Also discuss the importance of surveys and what they tell us, and how the results might be used (as the last question hints).

STUDENT SURVEY

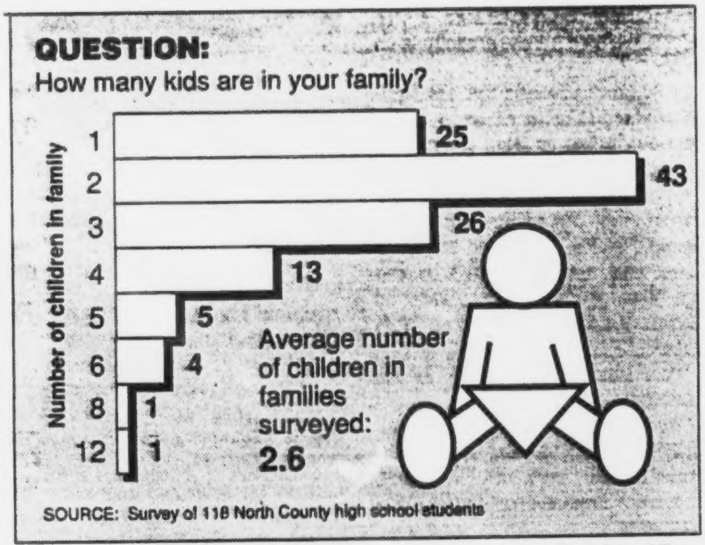
1. Compose a survey of three questions, asking 100 middle school students at your school their personal opinions on an issue important to them and/or statistical information about themselves (see example). Surveys may be conducted orally or in written form. Use a pictorial graph to represent your data for each of the three questions.

2. Make sure your survey reflects a fair sampling of the different populations found at your school. Consider all the ethnic groups and survey a proportional number of students from each group.

3. Write a brief summary stating any generalizations or conclusions drawn from your data. Present your data and summary to the class.

4. Can you project how a larger population would respond to the same questions. Consider the entire school population and school district population of middle school students. How would the numbers change? Would the pictorial graphs look differently? Explain.

Student survey



SOURCE: Survey of 118 North County high school students

The High School Page

THE QUESTION IS. . .

FOCUS: Application Activity

PURPOSE: The student will . . .
- Determine from a numerical answer based on given data the question that would result in that answer;
- Write the equation that would lead to the given answer;
- Provide the meaning for the operation used in each case, if this has been included in instruction;
- Use information provided to write entire word problems with prescribed characteristics;
- Solve the problems they write; and
- Solve the problems written by another student, group, or class.

STUDENT BACKGROUND: Students should have some experience in writing word problems. They should be familiar with the varied meanings of the arithmetic operations as included in the answers below.

MATERIALS: THE QUESTION IS. . . worksheet, calculators.

LESSON DEVELOPMENT: Students may work in groups for both parts of the lesson. In Part 1, students use the information given to decide what question could be asked to give the answer in each case. Explain to the students that any reasonable question is acceptable and that questions that require more than one step will require an equation and an operation meaning for each step. Students may need an example or review of operation meanings. For Problem 3, students could be reminded that there are 4 quarts in a gallon and 4 cups in a quart--to guide them toward the suggested multistep question. If necessary, remind students that 40% = 2/5 (Problem 7). Each group's responses should be shared and discussed in class.

In Part 2, students use the information from the same situation to write problems. If needed, review the meanings of the operations. Remind students that they should provide an answer key for their problems. Each group may work the problems of another group and during class discussion of the problems, suggest the best ones for a problem set to be given to another similar class to work.

ANSWERS:

Part 1: Other problems are possible. Discuss other reasonable questions students suggest for the answers given.

1. How much will the ice cream for the party cost?
   7 x $2.89 = $20.23; Equal-groups multiplication or Rate
2. How much will you have to pay your brother to work for 1 1/2 hours?
   1.5 x $2.50 = $3.75; Rate multiplication
3. How many scoops of ice cream can be made from one-half gallon?
   1/2 x 4 quarts = 2 quarts; Part-of-a-whole multiplication
   2 quarts x 4 cups = 8 cups; Equal-groups multiplication
   8 cups ÷ 1/2 = 16; Repeated subtraction
4. How many different single-scoop cones can be made with no topping?
   5 x 2 = 10; Making-choices multiplication
5. How many scoops of vanilla (or chocolate) can you serve?
   2 x 16 = 32; Equal-groups multiplication
6. What part of the ice cream is vanilla?
   2/7 x 7 = 2; Missing-factor: Part of a group.
7. What percent of the flavors contain no chocolate?
   p x 5 = 2;
8. How many scoops could each person have, if the ice cream were divided equally?
   7 x 16 = 112; Equal-groups multiplication or Rate
   112 ÷ 28 = 4; Sharing equally division

Part 2: Answers vary

SOURCE: Adapted from
Cook, M. (1990). *If. . . then. . . think & think again.* Balboa Island, CA: Marcy Cook.

THE QUESTION IS. . .

Ice cream cones will be served at your next party. You have two kinds of cones for your 27 guests to choose from, sugar and regular. You have asked the guests about their favorite flavors of ice cream, and as a result have bought five flavors:

2 half-gallons of vanilla
2 half-gallons of chocolate
1 half-gallon of chocolate chip
1 half-gallon of strawberries and cream
1 half-gallon of cookies and cream.

Each half-gallon of ice cream costs $2.89.

You will have nuts, chocolate, caramel, and strawberry sauces, and marshmallow creme available for toppings.

You have hired your younger brother at $2.50 per hour to make the ice cream cones as your guests request them. He will use half-cup scoops for the cones.

Part 1

Below are the answers to some questions about this situation. For each,

a) Write a question for which it **could** be the answer;
b) Write an equation that represents the problem; and
c) Give the meaning of the arithmetic operation as used in the problem.

1. $20.23

2. $3.75

3. 16

4. 10

5. 32

6. $\frac{2}{7}$

7. 40%

8. 4 (Write a question that uses the fact that there are 28 people eating ice cream at the party.)

Part 2

Based on the information in Part 1, write more questions you could ask. Most of your questions should require more than one step to answer. Write at least one question of each of the following kinds:

a) Rate
b) Making choices
c) Comparison subtraction
d) Equal-groups-or-amounts multiplication
e) Part-of-a-whole multiplication
f) Sharing-equally division
g) Repeated-subtraction division

THIS IS YOUR LIFE

FOCUS: Situational Activity

PURPOSE: The student will...

- Learn to budget a monthly income;
- Learn to make appropriate choices given a budget;
- Learn to use a checking account; fill out a transaction register and checks;
- Calculate a monthly income from hourly and annual wages and/or from a combination of jobs; and
- Imitate a real-life situation.

STUDENT BACKGROUND: Students should possess decimal number sense and be computationally competent with decimals.

MATERIALS: THIS IS YOUR LIFE worksheet, transaction register, checks, and calculator.

LESSON DEVELOPMENT: The occupations can be assigned randomly. Supplement those given with occupations that are common in your particular community. The first task is to figure monthly income. Teacher may want to demonstrate for the class how to fill out a transaction register and a check correctly and completely. All other assistance should be given individually. Students should be encouraged to help each other. The ending balance on the worksheet should equal the balance on the transaction register (after all the deductions) minus the cash items (ending balance on worksheet = balance on transaction register - cash items). Students are required to live within their means, and must budget accordingly. Students may want to exchange work and check for errors if balance is off.

ANSWERS: Salaries: waiter/waitress: $1610/mo.; clerk/secretary: $1020/mo.; teacher: $1908.33/mo.; student: $1440/mo.; computer programmer: $2160/mo.

THIS IS YOUR LIFE

You are going to assume an occupation, income, and all responsibilities of an adult living on his/her own. Calculate your monthly income and select one choice/plan for each necessity. Make sure to stay within your budget. All expenses of necessity are paid for by check. Fill out the worksheet before writing any checks. Fill out the transaction register as you write checks. If money remains, there are options you may choose from for extra expenses.
From the lists below make a choice for each of the following necessities.

- Food Plan
- Transportation Mode
- Housing
- Phone Plan
- Savings Plan

FOOD

Economy meals- $160/month
"Regular" meals- $240/month
Gourmet meals- $280/month

Make checks payable to VON'S FOOD STORE.

TRANSPORTATION

Economy car- per month: $150, $50 insurance, $25 gasoline
Midsize car- per month: $200, $80 insurance, $40 gasoline
Luxury car- per month: $300, $100 insurance, $40 gasoline
Bus Pass- per month: $50

Make checks payable to MIDLAND CORPORATION or if by bus COUNTY TRANSIT AUTHORITY.
Insurance is paid to FARMER'S INSURANCE.
Gasoline is paid to MOBIL OIL CO.

HOUSING RENTALS and UTILITIES

STUDIO

1 room, per month: $400, $40 electricity, $20 water
Make checks payable to Green Meadow Terrace Apartments.

1 BEDROOM

Per month: $600, $50 electricity, $20 water
Make checks payable to Malibu Terrace Apartments.

CONDOMINIUM

Pool, tennis courts, spa. Per month: $750/month, $70 electricity, $25 water
Make checks payable to Beverly Hills Condominiums.

Make checks payable to the apartment complex you choose.
Electricity is paid to __________ GAS & ELECTRIC. (fill in city)
Water is paid to CITY OF ____________. (fill in city)

PHONE

Option 1 - only local calls: $8.50/month.
Option 2 - local calls plus two long distance calls: $15/month.
Option 3 - local calls plus three long distance calls: $20/month.

Make checks payable to PACIFIC BELL.

SAVINGS

Plan 1 - Save $20/month, earn $0.10 interest/month
Plan 2 - Save $50/month, earn $0.25 interest/month
Plan 3 - Save $100/month, earn $0.50 interest/month

Deduct your savings deposit and add the interest to checking account; no check necessary.

EXTRA OPTIONS

CLOTHING

	SEARS	PENNEY'S	BROADWAY
Pants	$15	$20	$30
Shirts	$12	$15	$23
Shoes	$28	$35	$50
misc.	$7	$10	$15

Make checks payable to the department store.

ENTERTAINMENT

Cash items.

AMC Movie Theatre	$4.00
Palomar Bowling Lanes	$5.00
Golf Land	$3.00
Disneyland	$40.00
Magic Mountain	$30.00
San Diego Padres Game	$10.00

DINING OUT

Cash items. Sit down - $8.00 Fast food - $5.00

WORKSHEET

NECESSITIES:
Food ____________________
Car ____________________
Insurance ________________
Gasoline _________________
Housing __________________
Electricity _______________
Water ___________________
Phone ___________________
Subtotal 1 ______________

OPTIONS:
Check items:
____
____
____
____
Cash items:
____
____
____
____
SUBTOTAL 2 _______________

BUDGET: _____________
Savings (deduct)__________
Balance = _____________
Subtotal 1 (deduct)_______
Balance = _____________
Subtotal 2 (deduct) _______
Balance = _____________
Interest (add) ___________
Ending Balance ________

OCCUPATIONS

WAITER/WAITRESS
$3.50/hr. $8.00 tips/hr.; 35 hrs/week

CLERK/SECRETARY
Clerk - $5.50/hr.; 20 hrs/week
Secretary - $7.25/hr.; 20 hrs/week

TEACHER
$22,900 annual salary

COMPUTER PROGRAMMER
$13.50/hr.; 40 hrs./week

PART-TIME STUDENT
Construction worker
$18/hr.; 20 hrs./week

TRANSACTION REGISTER

* USE THESE CODES WHEN RECORDING YOUR NON-CHECK TRANSACTIONS
DEPOSIT = D DEBIT CARD = DC HOME COMPUTER = HC MACHINE = M PHONE = PH OTHER = O

* ITEM # OR TRANS CODE	DATE	DESCRIPTION OF TRANS	PAYMENT (-)	DEPOSIT (+)	FEE (-)	✓ TAX	BAL FORWARD $	

[redacted]

1065

90-7001/3222

__________ 19 ___

Pay to the order of __________ $

__________ Dollars

HOME FED BANK, FSB
CARLSBAD OFFICE
2580 EL CAMINO REAL
CARLSBAD, CALIFORNIA 92008

____

[redacted] [redacted]

[redacted]

1065

90-7001/3222

__________ 19 ___

Pay to the order of __________ $

__________ Dollars

HOME FED BANK, FSB
CARLSBAD OFFICE
2580 EL CAMINO REAL
CARLSBAD, CALIFORNIA 92008

____

[redacted] [redacted]

[redacted]

1065

90-7001/3222

__________ 19 ___

Pay to the order of __________ $

__________ Dollars

HOME FED BANK, FSB
CARLSBAD OFFICE
2580 EL CAMINO REAL
CARLSBAD, CALIFORNIA 92008

____

[redacted] [redacted]

[redacted]

1065

90-7001/3222

__________ 19 ___

Pay to the order of __________ $

__________ Dollars

HOME FED BANK, FSB
CARLSBAD OFFICE
2580 EL CAMINO REAL
CARLSBAD, CALIFORNIA 92008

____

[redacted] [redacted]

WATER CONSERVATION

FOCUS: Situational Activity

PURPOSE: The student will...

- Use mathematics in a natural setting;
- Read research to learn;
- Write a report to learn;
- Give an oral report to communicate what has been learned; and
- Use charts, graphs, or tables.

STUDENT BACKGROUND: Students need to know how to read and create charts, graphs, and/or tables. They should also be computer literate, especially with word processing.

TEACHER BACKGROUND: Too often students can do simple computations or even be able to perform complex operations, without really knowing about real-life, natural setting kinds of applications. This lesson presents a current situation as a problem with an immediate need for a genuine solution. It is an opportunity to solve a problem and really know it works when math is used as a tool of analysis and evaluation.

MATERIALS: WATER CONSERVATION activity sheet, water conservation kit, and a calculator.

LESSON DEVELOPMENT: Two weeks before you plan to use this lesson, have a student write to: San Diego Water Authority, 3211 Fifth Avenue, S.D., CA, 92103. In that letter, have the student tell the water authority what your plans are and that you need a class set of their water conservation kits.

The week before you plan to begin the assignment send a communication home to the parents asking for permission (and their assistance) in conducting a water conservation effort in conjunction with a math homework assignment. This is especially important since there may be some special home circumstances that do not accommodate this kind of homework assignment. Any students in this situation can be grouped with other students to collaborate with them, using their data.

Before the assignment, have the students collect all the current events they can that might involve water issues in San Diego. Then, have the students present their clippings as you facilitate a discussion. End it with your presentation of the Water Conservation Kit.

With the ground work laid, the assignment is ready to begin. Inform the students that they will need to conduct the experiment for one week. At the end of the experiment, they will write an essay, and present their findings to the class. Be sure to provide ample time for all students/groups to give 5-10 minute oral presentations.

Students should be encouraged to work in groups.

ANSWER: Will vary.

WATER CONSERVATION

Conserving water in San Diego is a current and pressing social, economic, and political issue. The San Diego Water Authority has developed a kit and "24 things" every citizen can do to save water. Your assignment is to see if you can really conserve. Use the kit's tips as well as any you can come up with. Be ready to show in a classroom exhibition how you have achieved your goal.

Write a three page type-written paper that details your analysis, and create any charts, tables, or graphs that will assist you with your oral presentation.

WORST T-SHIRT EVER

FOCUS: Situational Activity

PURPOSE: The student will . . .

- Research the financial realities of parenthood;
- Consider other uses of mathematics in parenting;
- Come to recognize the fallacy of the T-shirt message and gain some appreciation for the importance of mathematics in everyday life;
- Understand, through research, the gender-equality-in-mathematics issues implicit in the T-shirt message; and
- Express the results of their work in group reports and in letters to the T-shirt retailer.

TEACHER BACKGROUND: Since this is a situational lesson, the teacher should provide as little direction as is possible while reaching two loosely defined goals: students realize the fallacy of the T-shirt message for parents generally and the negative effects of such attitudes on females specifically.

STUDENT BACKGROUND: If students have previously used the lesson, This Is Your Life, they are familiar with some facets of living within a budget.

MATERIALS: WORST T-SHIRT EVER advertisement (ditto for each student or use as an overhead)

LESSON DEVELOPMENT: Students are to discuss the message of the T-shirt and, through research and discussion reject it, ultimately. The following suggestions may be used as desired if students need teacher guidance.

If the class has previously used a lesson on budgeting, they could consider the additional amount of money per month needed to raise a child--and the one-time expenses involved. Different groups could consider the financial impact of children of different ages. Encourage the students to research, including discussing the issue with parents, the other ways mathematics is important in parenting, although the financial aspects may be primary. They could pool this information, sharing it with the group working on the age group for which it is appropriate.

Students can bring to class, or be provided with, newspapers and catalogs for finding ads for the needs of the children in the age group they are researching. The teacher may be a resource in suggesting aspects they fail to consider, doctor bills, music and other lessons, reading temperature scales, instructions for dispensing medicine, ... and sources for information (such as NOW). Require enough actual use of mathematics by students in their the work during this activity to support the conclusion that parenting requires mathematics. They should identify, pose, and solve mathematics problems that could arise in parenting.

Students should realize that parenting is an extremely important and difficult, as well as rewarding, activity. They could be asked to use either a ten-pound sack of flour or an uncooked egg to represent a baby for some specified period of time, such as a week. Students should be responsible for the care of this "baby" at all times during the time period, making provisions for baby sitting when they are unable to care for it personally. Role playing situations could be set up by students as part of their group presentations, illustrating uses of mathematics in varied situations.

One group could research the contribution of the attitude conveyed by the T-shirt to women's earning power relative to that of men, single mothers living in poverty compared to men, etc. *Math Equals* would be useful to this group. It presents positive images of women in mathematics and provides evidence for the equal ability of females in mathematics and its equal importance in their lives (since most will need to work outside the home and the power of choice in occupation requires education). A few pages of this publication are included here. Other relevant publications are listed in the Resources.

The sexist bias in the T-shirt slogan should be seen as no more humorous and no more acceptable than racial slurs--the attitude is too pervasive and too harmful to be funny. The teacher might suggest that the group design and conduct attitude surveys of various groups about the relative importance of mathematics for males and females. Results of such surveys in the NAEP could be presented as a basis for this activity. Here is an example.

Mathematics is more for boys than girls.

		Percent Strongly Agree or Agree		
	Age	1978	1982	1986
Nation	13	2.5	3.3	6.1
	17	2.2	2.4	3.1
Male	13	3.4	4.3	6.2
	17	3.0	3.3	4.2
Female	13	1.7	2.3	6.1(sic)
	17	1.5	1.5	2.0

Although only a small percentage of those in each age group hold the views that mathematics is "more for boys than girls," in 1986, "significantly more 13-year-old females agreed with this statement than in either 1978 or 1982. The overall response pattern for males and females at ages 13 and 17 indicates a slight movement toward increased sex-role stereotyping."
"Recent findings that a lower percentage of females than males believe that they are good in mathematics, together with a slight increase in sex-role stereotyping, may be cause for concern and attention" (Dossey, et al., 1988, pp. 98-99).

The final product from each group should include a class report and a scathing (if unsent) letter to the T-shirt retailer explaining the importance of mathematics to parents and the harm to women from the attitude the shirt conveys.

EXTENSIONS: Based on their research, students could design a more appropriate T-shirt. They might research having it made for sale (by themselves or others) to promote the equality of women in mathematics.

Student of both genders might be interested in investigating racial imbalance in the field of mathematics.

References:
Dossey, J. A., Mullis, I. V. S., Lindquist, M. M., & Chambers, D. L. (1988). *The mathematics report card: Are we measuring up?* Princeton, NJ: Educational Testing Service.

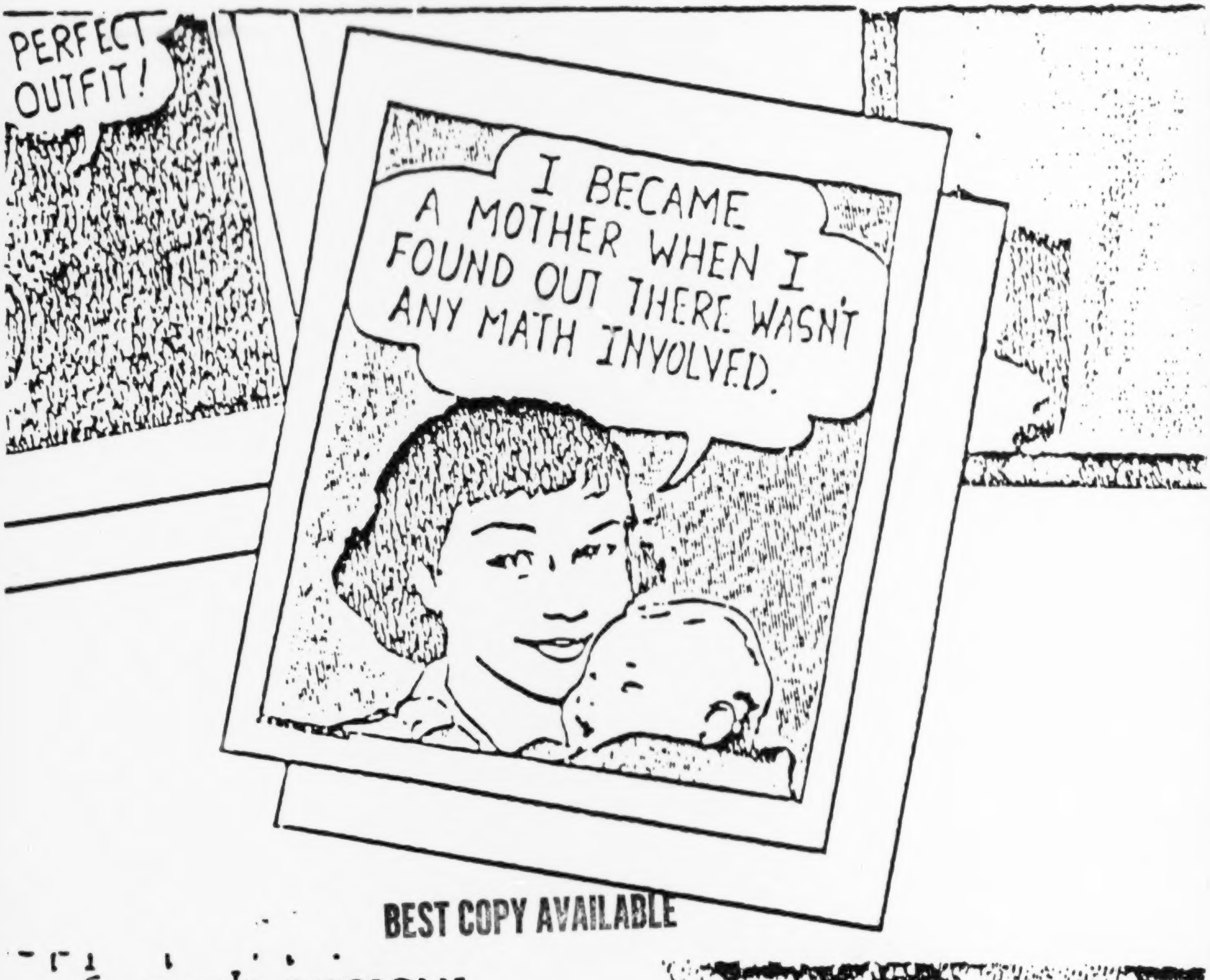
Additional suggested references:
Downie. D., Slesnick, T., & Stenmark, J. K. *Math for girls and other problem solvers.* (Grades 1-6). Available through Dale Seymour

Lawrence Hall of Science. *Spaces.* Available through Dale Seymour or EQUALS (Lawrence Hall of Science, University of California, Berkeley, CA 94720) (Includes a chapter entitled "Women in Careers")

Perl, T. (1978). *Math equals: Biographies of women mathematicians + related activities.* Menlo Park, CA: Addison-Wesley. (Grade 6 and up)

Skolnick, J., Langbort, C., & Day, L. *How to encourage girls in math and science*. (Teachers, Grades K-8) Available through Dale Seymour

Perhaps your school district has *Multiplying Options and Subtracting Bias* (1981), directed by Elizabeth Fennema. It is "a videotape and workshop intervention program designed to eliminate sexism from mathematics education." The program includes four videotapes, one directed at each of the following audiences: students, parents, teachers, guidance counselors. The videos are available from NCTM for $125 each, including a guide for incorporating the tapes into a workshop.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I BECAME A MOTHER WHEN I FOUND OUT THERE WASN'T ANY MATH INVOLVED.

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